# Supplementary Appendix for: de la Cuesta, Brandon, Naoki Egami, and Kosuke Imai. "Improving the External Validity of Conjoint Analysis: The Essential Role of Profile Distribution." *Political Analysis*

## A Review of Conjoint Literature

A review of the literature was conducted to assess several features of current best practices. In order to gather a sufficiently large number of articles, we selected 10 journals for a keyword search ("conjoint"): *The American Journal of Political Science, The American Political Science Review, The British Journal of Political Science, Journal of Experimental Political Science, Journal of Politics, Political Analysis, Political Behavior, Political Science Research and Methods, Research and Politics*, and *the Review of International Organizations.* This search criteria resulted in a total of 40 articles. We then augmented this list by examining articles that cited Hainmueller *et al.* (2014) using Google's "cited by" feature to obtain articles from additional journals or articles from the list above that were missed in the keyword search. This resulted in an additional 25 articles. We removed from the list any article whose contribution was primarily or completely methodological. This procedure left us with a total of 59 articles from 2014 to 2019. This list is not meant to be exhaustive but rather to be broad enough to give an overview of current practice.

Each article was then examined and classified along several dimensions. First, we coded the randomization distribution used in the design, characterizing each article by the number of factors used in the fielded design and the number that were randomized according to the uniform. In many cases, authors made no mention of the exact randomization probabilities or simply noted that their designs were "fully randomized". In cases where there was ambiguity about the distribution used, we consulted the appendix material to determine the distribution. If the appendix did not contain information sufficient to determine the distribution, we examined the uniformity of the standard errors of reported estimates and counted a factor as being randomized according to the uniform if the standard errors of all of that factor's levels were indistinguishable from each other.

We then examined the main text to establish whether the authors justified the distribution they chose on theoretical grounds. Justifications that would yield an affirmative coding include explicit discussion of the desire to match population distributions or the desire for statistical efficiency. An affirmative coding was given even if the discussion was relegated to a footnote and concerned only a single factor. Discussion of the constraints placed on unrealistic factor combinations was not part of the criteria used. As such, some papers that use such constraints were nonetheless considered as not invoking a substantive or theoretical justification for their chosen distribution.

Finally, for each paper we examined all factors that were a part of the design and determined whether it was feasible to collect data that would allow the approximation of the population distribution for that factor. Designs in which population data could be feasibly collected for most or all factors were considered to be amenable to the use of population data in the design or analysis stage. The articles are given below in Table A1 .

| **Author** | **Year** | **Title** |
|---|---|---|
| Atkeson and Hamel | 2018 | Fit for the job: candidate qualifications and vote choice in low information elections |
| Auerbach and Thachil | 2018 | How clients select brokers: competition and choice in India's slums |
| Ballard-Rosa et al | 2017 | The structure of American income tax policy preferences |
| Bansak et al | 2016 | How economic, humanitarian, and religious concerns shape European attitudes toward asylum seekers |
| Bechtel and Scheve | 2013 | Mass support for global climate agreements depends on institutional design |
| Bechtel et al | 2017 | Interests, norms and support for the provision of global public goods: The case of climate co-operation |
| Bechtel et al | 2017 | Policy design and domestic support for international bailouts |
| Berinksy et al | 2018 | Attribute affinity: U.S. natives' attitudes towards immigrants |
| Bernauer et al | 2019 | Do citizens evaluate international cooperation based on information about procedural and outcome equality? |
| Breitensten | 2019 | Choosing the crook: a conjoint experiment on voting for corrupt politicians |
| Bueno | 2017 | Bypassing the enemy: distributive politics, credit claiming, and nonstate organizations in Brazil |
| Campbell et al | 2016 | Legislator dissent as a valence signal |
| Carnes and Lupu | 2016 | Do voters dislike working-class candidates? Voter biases and the descriptive underrepresentation of the working class |
| Chauchard | 2016 | Unpacking ethnic preferences: theory and micro-Level evidence from north India |
| Chilton et al | 2017 | Reciprocity and public opposition to foreign direct investment |
| Clayton et al | 2019 | Exposure to immigration and admission preferences: evidence from France |
| Crowder-Meyer et al | 2018 | A different kind of disadvantage: candidate race, cognitive complexity, and voter choice |
| Eggers et al | 2017 | Corruption, accountability and gender: do female politicians face higher standards in public life |
| Franchino and Segatti | 2019 | Public opinion on the Eurozone fiscal union: evidence from survey experiments in Italy |
| Franchino and Zucchini | 2014 | Voting in a multidimensional space: a conjoint analysis employing valence and ideology attributes of candidates |

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|---|---|---|
| Gallego and Marx | 2016 | Multi-dimensional preferences for labour market reforms |
| Goggin et al | 2019 | What goes with red and blue? Mapping partisan and ideological associations in the minds of voters |
| Hainmueller and Hopkins | 2015 | The hidden American immigration consensus: a conjoint analysis of attitudes towards immigrants |
| Hainmueller et al | 2015 | Validating vignette and conjoint survey experiments against real-world behavior |
| Hankinson | 2018 | When do renters behave like homeowners? High rent, price, anxiety, and NIMBYism |
| Hartman and Morse | 2018 | Violence, empathy and altruism: evidence from the Ivorian refugee crisis in Liberia |
| Hausermann et al | 2019 | The politics of trade-offs: studying the dynamics of welfare state reform with conjoint experiments |
| Heinric and Kobayashi | 2017 | Sanction consequences and citizen support: a survey experiment |
| Heinric and Kobayashi | 2018 | How do people evaluate foreign aid to nasty regimes? |
| Hemker and Rink | 2017 | Multiple dimensions of bureaucratic discrimination: evidence from German welfare offices |
| Horiuchi et al | 2018 | Measuring voters multidimensional policy preferences with conjoint analysis: application to Japans 2014 election |
| Horiuchi et al | 2018 | Identifying voter preferences for politicians' personal attributes: a conjoint experiment in Japan |
| Huff and Kertzer | 2017 | How the public defines terrorism |
| Iyengar and Westood | 2015 | Fear and loathing across party lines: new evidence on group polarization |
| Karpowitz et al | 2017 | How to elect more women: gender and candidate success in a field experiment |
| Kertzer et al | 2019 | How do observers assess resolve? |
| Kirkland, Coppock | 2018 | Candidate choice without party labels |
| Leeper, Robison | 2018 | More important, but for what exactly? The insignificant role of subjective issue importance in vote decisions |
| Li and Zeng | 2017 | Individual preferences for FDI in developing countries: experimental evidence from China |
| Liu | 2018 | The logic of authoritarian political selection: evidence from a conjoint experiment in China |
| Malhotra and Newman | 2019 | Explaining immigration preferences: disentangling skill and prevalence |
| Mares and Visconti | 2019 | Voting for the lesser evil: evidence from a conjoint experiment in Romania |
| Mummolo | 2016 | News from the other side: how topic relevance limits the prevalence of partisan selective exposure |
| Mummolo and Nall | 2016 | Why partisans do not sort: the constraints on political segregation |

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|---|---|---|
| Newman and Malhotra | 2018 | Economic reasoning with a racial hue: is the immigration consensus purely race neutral? |
| Oliveros and Schuster | 2018 | Merit, tenure, and bureaucratic behavior: evidence from a conjoint experiment in the Dominican Republic |
| Ono and Burden | 2018 | The contingent effects of candidate sex on voter choice |
| Ono and Yamada | 2018 | Do voters prefer gender stereotypic candidates? Evidence from a conjoint survey experiment in Japan |
| Peterson | 2017 | The role of the information environment in partisan voting |
| Peterson and Simonovitis | 2018 | The electoral consequences of issue frames |
| Sances | 2018 | Ideology and vote choice in U.S. mayoral elections: evidence from Facebook surveys |
| Scheider | 2019 | Euroscepticism and government accountability in the European Union |
| Sen | 2017 | How political signals affect public support for judicial nominations: evidence from a conjoint experiment |
| Shafranek | 2019 | Political considerations in nonpolitical decisions: a conjoint analysis of roommate choice |
| Spilker et al | 2016 | Selecting partner countries for preferential trade agreements: experimental evidence from Costa Rica, Nicaragua, and Vietnam |
| Teele et al | 2018 | The ties that double bind: social roles and women's underrepresentation in politics |
| Vivyan and Wagner | 2016 | House or home? Constituent preferences over legislator effort allocation |
| Ward | 2019 | Public attitudes towards young immigrant men |
| Write et al | 2016 | Mass opinion and immigration policy in the United States: re-assessing clientelist and elitist perspectives |

Table A1: Conjoint Articles Published From 2014-2019.

## B Constructing the Target Profile Distribution

We utilize several sources of data to construct the population distribution used in Section 2. We emphasize that ideally researchers should construct the population profile distribution before designing conjoint analysis in order to match the attributes of the population distribution with those of conjoint analysis. In the current application, we construct the population profile distribution after the conjoint analysis was conducted by Ono and Burden (2019). As a result, for almost all factors, additional *ex post* coding was needed to match the empirical data to the categories chosen by the original authors.

Here, we discuss the data source and the procedure used to produce categories matching those used in the original experiment. We use the legislators in the 115th Congress as the

| **Factors** | **Levels** | **Population Data Source** |
|---|---|---|
| Age | 36, 44, 52, 60, 68, 76 | Daily Kos Biographical Database |
| Gender | Male, Female | |
| Race | Asian, Black, Hispanic, White | |
| Family | Divorced, Never married, Married (no children) Married (2 children) | The Hill People Directory |
| Experience | None, 4 years, 8 years, 12 years | Daily Kos Biographical Database |
| Expertise | Economic policy, Education, Environmental issues, Foreign policy, Health care, Public safety (crime) | Congressional Committee Assignments |
| Character Trait | Compassionate, Honest, Intelligent, Knowledgeable, Leadership, Empathetic | None |
| Immigration Policy | Favors guest worker program, opposes guest worker program | Secure America's Future Act (SAFA) Roll Call Votes |
| Security Policy | Strong military, Cut defense spending | Center for Security Policy Legislator Scorecard |
| Abortion Policy | Pro-choice, Neutral, Pro-life | National Right to Life Council Legislator Scorecard |
| Deficit Policy | Increase taxes, Take no action, Reduce spending | Club for Growth Legislator Scorecard |

Table A2: Levels and Data Sources Used to Construct the Population Profile Distribution.

target population distribution in order to maximize our ability to reverse engineer the original factor levels. To merge disparate data sources, we use a probabilistic record linkage method, implemented via the `R` package `fastLink` (Enamorado *et al.*, 2019), with partial matching on the first and last name. Table A2 lists the data source used to build the empirical distribution for each factor. In calculating these factors, we considered only legislators who were seated via popular vote; those who were named to a seat due to a vacancy are omitted.

## B.1 Demographic Factors

Age, gender and race — the three demographic factors used in the original study — were obtained from the Daily Kos 115th Congress Members Guide. The dataset contains both biographical and electoral information. Biographical information on legislators is sourced from Pew, Roll Call, news stories and Wikipedia. Data was also checked against a similar dataset available through `legislatoR` (Gobel and Munzert, 2019), an `R` package that allows queries to a database of biographical and political information on legislators from multiple countries. The details about each demographic factor is presented below.

**Age.** The age factor was produced by binning legislators' ages into the same age ranges as the original categories.

**Gender.** Legislator gender was taken directly from the data and unaltered.

**Race.** Racial categories closely matched those of the experiment with some notable exceptions. First, legislators that were coded as identifying as both white and Hispanic were coded as Hispanic in the joint data. Two legislators who identified as white-Portuguese American were coded as White. All Asian-American legislators were coded as Asian-American regardless of their nationality. For example, an Indian-American and Japanese American legislator would both be coded as Asian-American.

### B.2 Background Factors

Background factors were constructed from four sources: the Daily Kos 115th Congress Members Guide, legislators' official Wikipedia page; the Congressional Committees dataset (Stewart and Woon, 2017); and biographical information from the People directory of TheHill.com, a digital news site.

**Experience.** The experience measure was created by first subtracting the first year a legislator was elected to higher office from the most recent election year, resulting in a measure of the total number of years spent in office. To calculate years served, the election year for all House members was taken as 2016—the year of the most recent House elections present in the data—while for Senators the election year in which they won current office was used. If a legislator had served previously, this interval was added to the more recent tenure. In a limited number of cases, a legislator was seated as a result of a special election. In these cases, the year of the special election is used. To map this measure onto the categories of the factor used in the original experiment (0 years, 4 years, 8 years, 12 years), we use the midpoints between each category to determine into which bin each observation falls. For example, a legislator with 1 year experience would fall to the left of the midpoint between the two nearest categories (0 and 4 years, respectively) and be assigned to the 0 years category.

**Policy Expertise.** The policy expertise factor is difficult to approximate with real-world data because the expertise that legislators claim during campaigns may be a matter of political expedience and may not correspond to their actual expertise. To overcome this difficulty, we used legislators' committee assignments as the basis for producing the joint distribution. Our motivation for using committee assignments is straightforward: legislators are strategic in their choice of committee assignments—or at least in their attempts to obtain assignments that would allow them to claim expertise in politically salient areas. Using the Congressional Committees dataset, we attempted to map each committee—in both the House and Senate—to a corresponding category in the original experiment. Where the committee was a poor match for all of the original categories, such as for the Ways and Means Committee, we coded a legislator's expertise as missing.

Because each legislator serves on multiple committees and our joint distribution is constructed at the legislator-level, there are multiple values possible for each legislator. To overcome this problem, for each legislator we compared the seniority rankings of each committee and assigned to that legislator the committee on which they were the most senior. Because not all committees are the same size, it is possible that a legislator could be assigned a small committee in which they were a higher rank in absolute terms but lower as a percentage of total seats. We allow for such cases because a high absolute ranking on a small committee may be used as the basis for a claim of expertise as easily as a lower ranking in a larger committee.

**Party.** Party was taken directly from the Daily Kos dataset and then binned into three categories: Democrat, Republican and Independent.

**Favorability Rating.** Because we were interested in a large pool of legislators, it was not possible to obtain favorability ratings drawn from a sufficiently large survey sample for the majority of our legislators. To overcome this, we used the vote margin in the previous election as a proxy for legislators' approval ratings in their constituency. Due to the mechanics of first-past-the-post elections, this means that the lowest level of favorability rating possible in the experiment (34%) occurs only twice and the next highest rating (43%) occurs only six times. This right-skewed distribution is a good approximation of the true favorability rating for two reasons. First, viable candidates in competitive districts must have reasonably high approval ratings with the general electorate. Second, legislators in stronghold districts are likely to have high approval ratings due to their copartisanship with the majority of their constituents.

**Family Status.** The original "family" factor included information on marital status and legislators' number of children. Data on both were harvested from legislators' Wikipedia pages using the `rvest` package in `R` (Wickham, 2019) wherever such fields were available. Because legislators may not wish to publicize that they are divorced or unmarried, it is possible that missingness is correlated with legislators' marital status. We attempted to address this problem by augmenting the Wikipedia data with data harvested from the People Directory of TheHill.com. For both the Wikipedia and TheHill.com fields, the names of spouses and number of children were given. In the case of multiple marriages, we coded legislators' marital status based on the status of their most recent marriage. Thus, a legislator who is currently married but was divorced in the past would be coded as married. To use this data to reconstruct the categories used in the conjoint experiment, it was necessary to bin the number of children into the original categories. Legislators with 1 or more children were binned into the "2 children" category. The number of children was disregarded if the legislator was divorced or never married because the original categories contained no information on the number of children for those marital statuses. Legislators for whom the number of children field was missing in both TheHill and Wikipedia datasets were coded as having zero children.

## B.3 Policy Positions

The original data contained information on four policy dimensions: abortion (pro-life/pro-choice/neutral); immigration (in favor of/against a guest worker program); security (favors strong military/favors defense spending cuts); and deficit reduction (wants to reduce deficit through tax increases/wants to maintain current deficit/wants to reduce deficit through spending cuts). These factors were difficult to approximate with real-world data for several reasons. First, they correspond to broad issue areas, such as a legislators' stance on abortion. In such cases, real-world legislators' policy positions are likely to be driven by one or more latent dimensions that can only be estimated from voting behavior across many bills. Second, estimates of this latent dimension via voting behavior are complicated by the fact that we are restricted to considering only bills introduced in and voted on during the 115th Congress, resulting in relatively few bills that correspond to the original levels. Third, for positions that are subsets of a broader policy space—such as the desired means of deficit reduction—a bill with a proposal similar to the original levels will often include statutes related to other, similar issues. Special care thus needs to be taken to ensure that legislators' voting behavior was driven at least in part by the statute corresponding to the original levels.

Finally, while policy think tanks often provide legislator scorecards, the score space may not correspond neatly to the levels of the original data. For example, someone who is considered moderate on fiscal issues may not necessarily advocate for no deficit reduction, which is the middle category of the spending policy factor. Given these considerations, we aimed to build a reasonable first approximation using a combination of actual voting behavior. In cases where legislators' voting behavior was not available or driven by other statutes included in a bill, legislator scorecards produced by advocacy organizations and partisan policy institutes were consulted.

To facilitate consistency across the four policy factors in the original data, we used a general heuristic in deciding whether to use a bill or a legislator scorecard to approximate the experimental categories. We began by identifying legislative scorecards whose policy space closely matched the policy referenced in the original factor. If none were available, a scorecard for a more broad issue area could also be used.

Legislator scorecards are typically constructed by "scoring" legislators' votes on bills that are considered important in a particular policy space. The think tank producing the scorecard then rates each legislator according to how closely their voting behavior matches the position favored or advocated by the organization. While such scorecards are available from both conservative and liberal policy institutes, we chose only from scorecards issued by conservative organizations. This was done to ensure that a higher score was always associated with a more conservative policy position. Once the scorecard was obtained, we examined the bills used to produce each legislators' score. If a bill closely matched the original categories and was voted on in the 115th Congress, each legislators' vote was used to assign him or her a policy position.

To ensure that the policy position distribution was not driven by only a few legislators, bills meeting these criteria were only used if they were considered in both the House and Senate in similar form or the House alone.

If there existed no bill that was suitable for approximating actual legislators' values on the original factor, the legislator scorecards were used directly. To do so, legislators were binned into categories according to their numerical score and normalized to range from 0 to 1. A score of 1 was given to legislators considered strong proponents of a given policy position. If the original factor had only two categories — as in the case of the national security factor, for example — legislators with a value at or below the midpoint (a score of 0.5) were given the value corresponding to the liberal position, while those above were assigned to the more conservative category. In cases where the original factor had three categories, legislators with scores from 0 to 0.4 were given the liberal position, those with scores between 0.4 and 0.6 were given the moderate position, and those at 0.6 or above were given the conservative position.

Given these decision rules, we selected legislator scorecards for the abortion, deficit spending and national security factors and a single bill for the immigration factor. Below we describe the data source and coding rules used to produce categories matching those of the original study.

**Abortion.** Data for abortion position was based on the National Right to Life Council (NRLC) legislator scorecard. While the NRLC is a conservative, pro-life organization, similar scorecards from left-leaning organizations produce similar distributions owing to the highly polarized nature of abortion policy in the United States. The NRLC score is a 0-1, with 1 corresponding to a strongly pro-life legislator and a zero a strongly pro-choice legislator. Legislators between 0 and 0.4 were coded as pro-choice; those between 0.4 and 0.6 were coded as neutral; and those with a score greater than 0.6 were coded as pro-life. Predictably, there are only three neutral legislators according to this criteria, and the distribution is almost perfectly correlated with partisanship.

**Immigration.** Using the decision rules above, we chose a bill rather than a legislator scorecard, selecting the Secure America's Future Act (SAFA) to serve as our proxy for legislators' positions on the guest worker program. SAFA included in it a provision to abolish the existing H-2B guest worker program with a less generous and more restricted policy dubbed H-2C. While other provisions of the bill were politically important—such as protection for so-called Dreamers—the guest worker program was a prominent feature of the bill. This is the only case for the 115th Congress where an immigration bill including a program closely corresponding to the Ono and Burden levels was both introduced and voted on. Because a vote in favor of SAFA was a vote for the H-2C guest worker program (and thus against the existing H-2B system), a vote of yes was coded as opposition to a guest worker program and a vote of no as favoring a guest worker system. Legislators who did not vote were marked as missing.

**National Security.** Data for the military spending factor was based on the Center for Security Policy (CfSP) legislator scorecard. The CfSP scored a total of 19 bills to produce a 0-1 score where higher values represents more "pro-security" voting behavior. Legislators with a score less than 0.5 were binned into the "cut military spending" category while those above the cutoff were binned into the "maintain strong defense" category.

**Budget.** Data for the budget position was based on the Club for Growth's legislator scorecard. The Club for Growth, a conservative organization, rated legislators according to the organization's pro-deficit-reduction position, producing a scorecard with a range of 0-1 where higher values indicate more support for deficit reduction. Legislators with a score between 0 and 0.4 were given a value of "reduce deficit through tax increases", a liberal position; those with a score between 0.4 and 0.6 were given the moderate position "do not reduce deficit now"; and those with a score greater than 0.6 were given the conservative position "reduce deficit through spending cuts".

## C Robustness to the Choice of Profile Distribution

The original experiment design of Ono and Burden (2019) considers hypothetical political candidates. Thus, the ideal target profile distribution would be the real-world distribution of the attributes summarized in Table 1 for all candidates, not only sitting legislators. Unfortunately, because the original experiment was not designed with fidelity to the real-world distribution in mind, there are many factors for which it is practically impossible to gather corresponding real-world distributions of all candidates (e.g., character traits of candidates and favorability rating). As a result, in Section 3.4.1, we set our main target profile distribution to be politicians in the 115th Congress, for whom we were able to collect real-world distributions for most factors (as described in detail in Section B).

In this section, we use the model-based approach to investigate the robustness of the pAMCE estimates based on the 115th Congress (reported in Section 5.1) to alternative profile distributions based on candidate-level data. Although these candidate-level data do not include information for all factors used in the original experiment, we can incorporate a number of relevant candidates' characteristics. In particular, we rely on two publicly available datasets on candidate characteristics, DIME data set (Bonica, 2015) and the Reflective Democracy (RefDem) dataset,[9] to improve the profile distributions of three demographic variables. In addition, we use our substantive knowledge to explore different theoretically relevant profile distributions on policy dimensions.

**Data.** For the DIME data (Bonica, 2015), we consider all major party candidates that ran for Congress in the 2014 general election, the last year of the dataset's coverage. This yields 1148 candidates. In the RefDem data, we consider all major party candidates who ran for the House or Senate in 2018. This yields 911 unique candidates. We use the DIME data to

[9] This dataset is available at `https://wholeads.us/resources/for-researchers/`

replace the marginal distribution for number of years in office (`Experience`), and the RefDem data to replace the marginal distributions of race and gender (`Race, Gender`). Figure A1 visualizes this new distribution. Comparing to Figure 2, the two most notable differences are the larger proportion of white candidates — particularly for Democrats — and the higher incidence of candidates with "no experience," a natural consequence of considering challengers. In a second profile distribution, we also augment these new demographic marginal distributions with changes to the marginal distributions of policy positions, making them more extreme to reflect the fact that winning candidates are systematically more moderate than losing candidates. Figure A2 visualizes this second new distribution. On all policy dimensions, we made the policy positions slightly more extreme, and it can be clearly seen in positions on `Deficit`. Our goal is to assess the robustness of the pAMCE based on the 115th Congress to these alternative target profile distributions.

**Results.** Figure A3 shows the pAMCE estimates of being female in Ono and Burden (2019) with three different profile distributions. The first row represents the pAMCE estimates reported in Section 5.1. The second and third rows show the results based on the first alternative profile distribution (with new marginals for three demographic factors) and the second alternative distribution (with improved demographic factors + more extreme policy positions), respectively. Although the difference between the uAMCE (black estimates) and pAMCE estimates for Republican (red) and Democrats (blue) are large, the change across different alternative target profile distributions is small. This suggests that even though the target profile distribution based on the 115th Congress is different from the ideal political candidate-level profile distribution, the pAMCE estimates based on the 115th Congress are robust to theoretically relevant changes in profile distributions that better reflect candidate-level data.

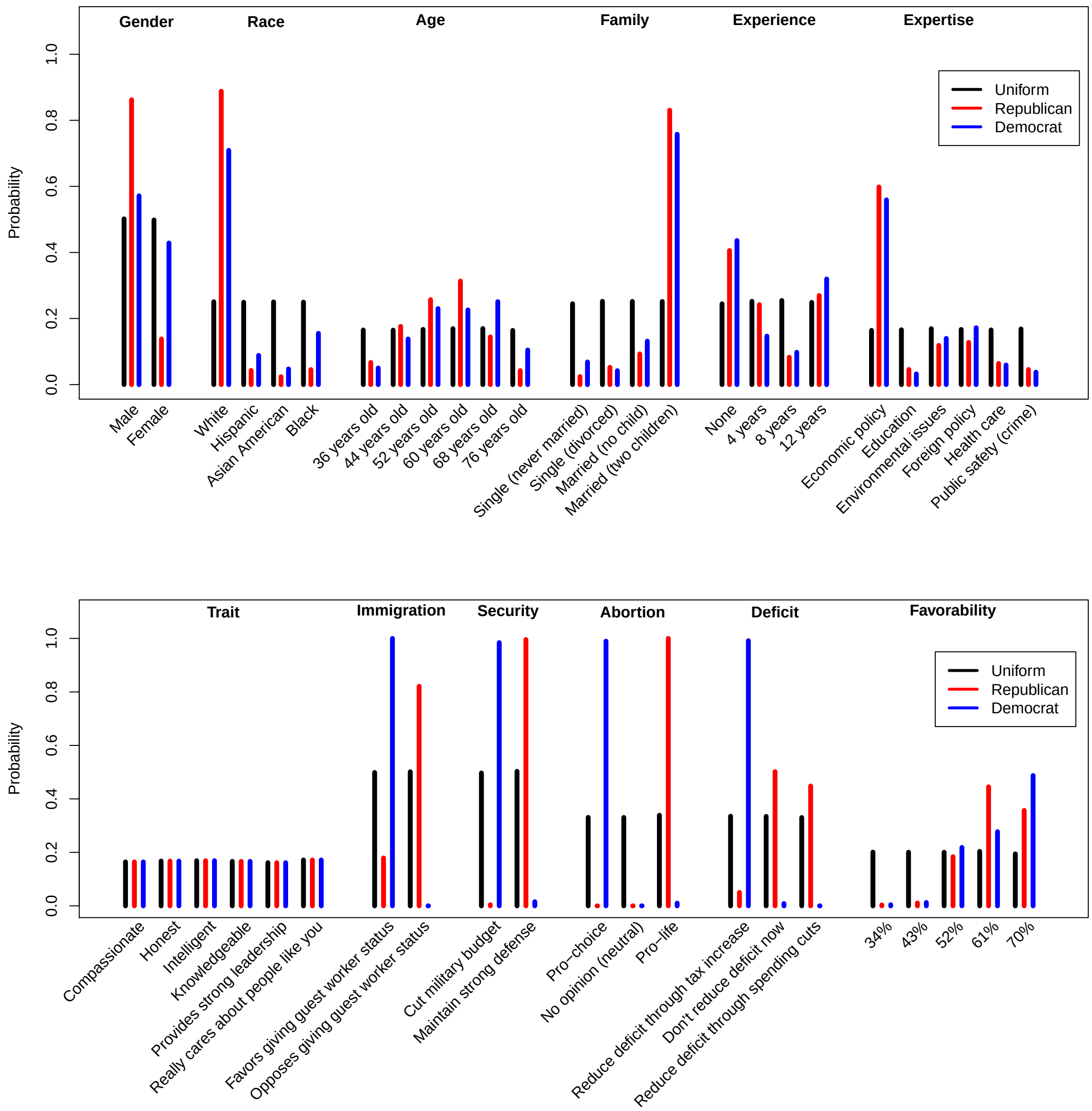
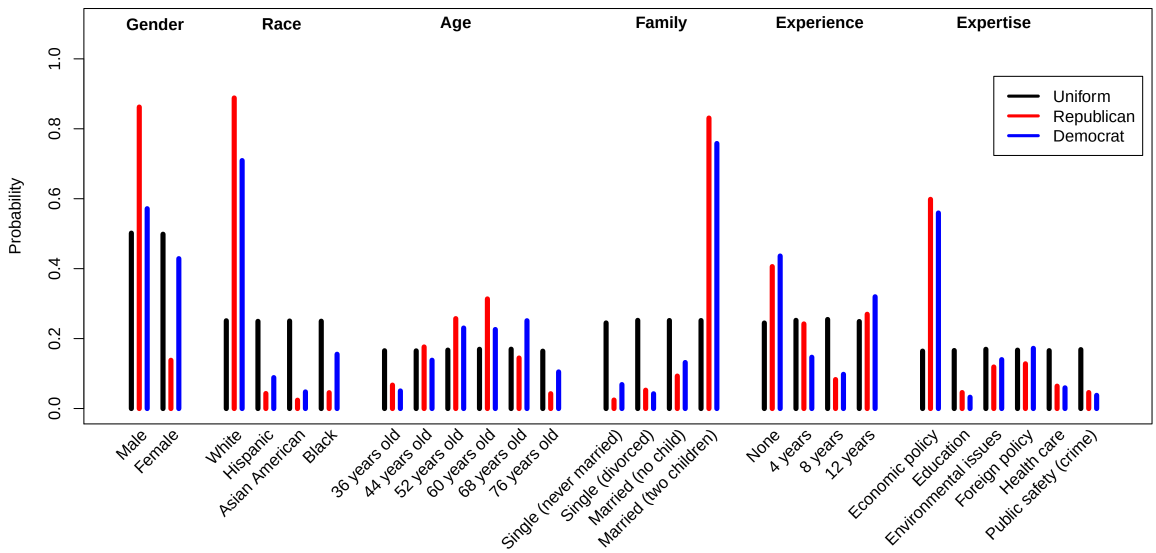



Figure A1: Experimental and Target Profile Distributions of Factors in Ono and Burden (2019) improved by candidate-level data sets.



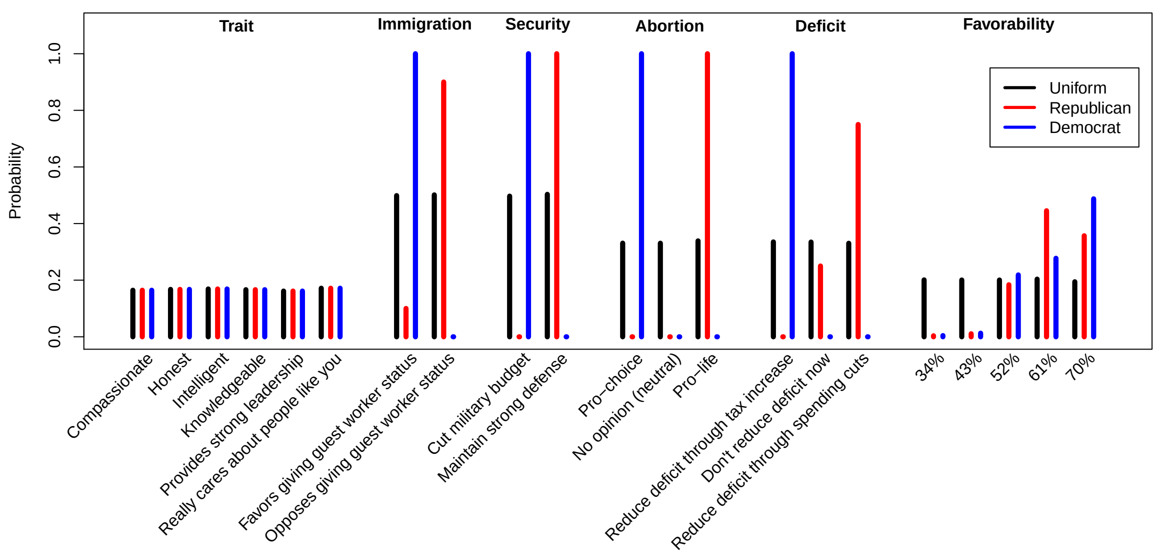




Figure A2: Experimental and Target Profile Distributions of Factors in Ono and Burden (2019) improved by candidate-level data sets and augmented with counterfactual more extreme policy positions.

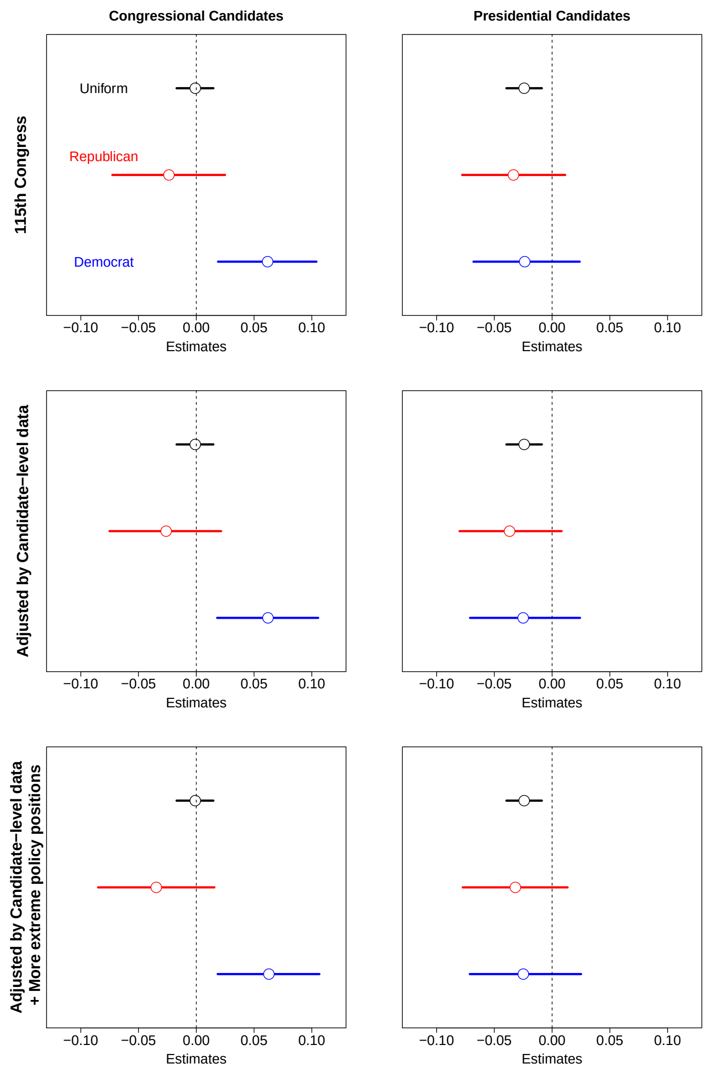



Figure A3: Estimates of the pAMCEs of Being Female in Ono and Burden (2019) with three different profile distributions. The first row represents the pAMCE estimates reported in Section 5.1. The second and third rows show results based on the first alternative profile distribution (with improved three demographic variables) and the second alternative profile distribution (with improved three demographic variables + more extreme policy positions).

# D Proofs

## D.1 Consistency of Weighted Difference-in-Means

Here, we formally prove that the proposed weighted difference-in-means estimator is consistent for the pAMCE under any randomization distribution that satisfies a set of positivity conditions.

Theorem 1 (Consistency of the Weighted Difference-in-Means Estimator) The weighted difference-in-means estimator defined in equation (5) is consistent for the pAMCE,

$$\widehat{\tau}_\ell^*(t_1, t_0) \xrightarrow{p} \tau_\ell^*(t_1, t_0), \tag{A1}$$

for any randomization distribution $\Pr^{\mathrm{R}}(\cdot)$ that satisfies the following positivity conditions,

$$\begin{aligned}
&\Pr^{\mathrm{R}}(T_{ijk\ell} = t_1 \mid (\mathbf{T}_{ijk,-\ell}, \mathbf{T}_{i,-j,k}) = \mathbf{t}) > 0\\
&\Pr^{\mathrm{R}}(T_{ijk\ell} = t_0 \mid (\mathbf{T}_{ijk,-\ell}, \mathbf{T}_{i,-j,k}) = \mathbf{t}) > 0\\
&\Pr^{\mathrm{R}}((\mathbf{T}_{ijk,-\ell}, \mathbf{T}_{i,-j,k}) = \mathbf{t}) > 0
\end{aligned}$$

for all $\mathbf{t} \in \mathcal{T}^*$ where $\mathcal{T}^*$ is the support of $\Pr^*(\mathbf{T}_{ijk,-\ell}, \mathbf{T}_{i,-j,k})$.

The positivity requirement guarantees that all possible profile combinations under the target population distribution have non-zero probabilities under the randomization distribution. The proposed three designs satisfy this requirement.

**Proof.** We want to prove that the following estimator is consistent for the pAMCE.

$$\widehat{\tau}_\ell^*(t_1, t_0) \;=\; \frac{\sum_{i=1}^N \sum_{j=1}^J \sum_{k=1}^K \mathbf{1}\{T_{ijk\ell} = t_1\} w_{ijk\ell} Y_{ijk}}{\sum_{i=1}^N \sum_{j=1}^J \sum_{k=1}^K \mathbf{1}\{T_{ijk\ell} = t_1\} w_{ijk\ell}} - \frac{\sum_{i=1}^N \sum_{j=1}^J \sum_{k=1}^K \mathbf{1}\{T_{ijk\ell} = t_0\} w_{ijk\ell} Y_{ijk}}{\sum_{i=1}^N \sum_{j=1}^J \sum_{k=1}^K \mathbf{1}\{T_{ijk\ell} = t_0\} w_{ijk\ell}},$$

where the weights are defined as,

$$w_{ijk\ell} \;=\; \frac{1}{\Pr^{\mathrm{R}}(T_{ijk\ell} \mid \mathbf{T}_{ijk,-\ell}, \mathbf{T}_{i,-j,k})} \times \frac{\Pr^*(\mathbf{T}_{ijk,-\ell}, \mathbf{T}_{i,-j,k})}{\Pr^{\mathrm{R}}(\mathbf{T}_{ijk,-\ell}, \mathbf{T}_{i,-j,k})}.$$

We first focus on the numerator. By the law of large number, we can obtain

$$\frac{1}{NK} \sum_{i=1}^N \sum_{k=1}^K \mathbf{1}\{T_{ijk\ell} = t_1\} w_{ijk\ell} Y_{ijk} \xrightarrow{p} \mathbb{E}[\mathbb{E}^{\mathrm{R}}[\mathbf{1}\{T_{ijk\ell} = t_1\} w_{ijk\ell} Y_{ijk}]],$$

where the first expectation is over a random sample of respondents $i$ and task positions $k$, and the second expectation is over randomization of treatment assignment. We focus on the expression inside the first expectation.

$$\begin{aligned}
&\mathbb{E}^{\mathrm{R}}[\mathbf{1}\{T_{ijk\ell} = t_1\} w_{ijk\ell} Y_{ijk}]\\
=& \sum_{(\mathbf{t}_{ijk,-\ell}, \mathbf{t}_{i,-j,k})} \mathbb{E}^{\mathrm{R}}[\mathbf{1}\{T_{ijk\ell} = t_1\} w_{ijk\ell} Y_{ijk} \mid \mathbf{T}_{ijk,-\ell} = \mathbf{t}_{ijk,-\ell}, \mathbf{T}_{i,-j,k} = \mathbf{t}_{i,-j,k}] \Pr^{\mathrm{R}}(\mathbf{T}_{ijk,-\ell} = \mathbf{t}_{ijk,-\ell}, \mathbf{T}_{i,-j,k} = \mathbf{t}_{i,-j,k})\\
=& \sum_{(\mathbf{t}_{ijk,-\ell}, \mathbf{t}_{i,-j,k})} \{\mathbb{E}^{\mathrm{R}}[\mathbf{1}\{T_{ijk\ell} = t_1\} w_{ijk\ell} Y_{ijk}(T_{ijk\ell} = t_1, \mathbf{T}_{ijk,-\ell} = \mathbf{t}_{ijk,-\ell}, \mathbf{T}_{i,-j,k} = \mathbf{t}_{i,-j,k}) \mid \mathbf{T}_{ijk,-\ell} = \mathbf{t}_{ijk,-\ell}, \mathbf{T}_{i,-j,k} = \mathbf{t}_{i,-j,k}]\\
&\qquad \times \Pr^{\mathrm{R}}(\mathbf{T}_{ijk,-\ell} = \mathbf{t}_{ijk,-\ell}, \mathbf{T}_{i,-j,k} = \mathbf{t}_{i,-j,k})\}\\
=& \sum_{(\mathbf{t}_{ijk,-\ell}, \mathbf{t}_{i,-j,k})} \{Y_{ijk}(T_{ijk\ell} = t_1, \mathbf{T}_{ijk,-\ell} = \mathbf{t}_{ijk,-\ell}, \mathbf{T}_{i,-j,k} = \mathbf{t}_{i,-j,k}) w_{ijk\ell}
\end{aligned}$$

$$\times\mathbb{E}^{\mathrm{R}}[\mathbf{1}\{T_{ijk\ell}=t_1\}\mid \mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell},\mathbf{T}_{i,-j,k}=\mathbf{t}_{i,-j,k}]\Pr{}^{\mathrm{R}}(\mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell},\mathbf{T}_{i,-j,k}=\mathbf{t}_{i,-j,k})\}$$

$$= \sum_{(\mathbf{t}_{ijk,-\ell},\mathbf{t}_{i,-j,k})}\{Y_{ijk}(T_{ijk\ell}=t_1,\mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell},\mathbf{T}_{i,-j,k}=\mathbf{t}_{i,-j,k})w_{ijk\ell}$$

$$\times\Pr{}^{\mathrm{R}}(T_{ijk\ell}=t_1\mid \mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell},\mathbf{T}_{i,-j,k}=\mathbf{t}_{i,-j,k})\Pr{}^{\mathrm{R}}(\mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell},\mathbf{T}_{i,-j,k}=\mathbf{t}_{i,-j,k})\}$$

$$= \sum_{(\mathbf{t}_{ijk,-\ell},\mathbf{t}_{i,-j,k})}\{Y_{ijk}(T_{ijk\ell}=t_1,\mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell},\mathbf{T}_{i,-j,k}=\mathbf{t}_{i,-j,k})\Pr{}^{*}(\mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell},\mathbf{T}_{i,-j,k}=\mathbf{t}_{i,-j,k})\},$$

where the first equality follows from the rule of conditional expectation, the second from the definition of potential outcomes, the third from the fact that potential outcomes and weights are fixed within the second expectation, the fourth from the definition of probability, and the final equality from the definition of weights we propose.

Due to the no profile-order assumption, we can average over $j$.

$$\frac{1}{NJK}\sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{k=1}^{K}\mathbf{1}\{T_{ijk\ell}=t_1\}w_{ijk\ell}Y_{ijk}$$

$$\xrightarrow{p}\mathbb{E}\left\{\sum_{(\mathbf{t}_{ijk,-\ell},\mathbf{t}_{i,-j,k})}Y_{ik}(T_{ijk\ell}=t_1,\mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell},\mathbf{T}_{i,-j,k}=\mathbf{t}_{i,-j,k})\Pr{}^{*}(\mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell},\mathbf{T}_{i,-j,k}=\mathbf{t}_{i,-j,k})\right\}.$$

For the denominator, we again use the law of large number.

$$\frac{1}{NK}\sum_{i=1}^{N}\sum_{k=1}^{K}\mathbf{1}\{T_{ijk\ell}=t_1\}w_{ijk\ell}\xrightarrow{p}\mathbb{E}[\mathbb{E}^{\mathrm{R}}[\mathbf{1}\{T_{ijk\ell}=t_1\}w_{ijk\ell}]].$$

Focusing on the expression inside the second expectation.

$$\mathbb{E}^{\mathrm{R}}[\mathbf{1}\{T_{ijk\ell}=t_1\}w_{ijk\ell}]$$

$$= \sum_{(\mathbf{t}_{ijk,-\ell},\mathbf{t}_{i,-j,k})}\mathbb{E}^{\mathrm{R}}[\mathbf{1}\{T_{ijk\ell}=t_1\}w_{ijk\ell}\mid \mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell},\mathbf{T}_{i,-j,k}=\mathbf{t}_{i,-j,k}]\Pr{}^{\mathrm{R}}(\mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell},\mathbf{T}_{i,-j,k}=\mathbf{t}_{i,-j,k})$$

$$= \sum_{(\mathbf{t}_{ijk,-\ell},\mathbf{t}_{i,-j,k})}\{w_{ijk\ell}\mathbb{E}^{\mathrm{R}}[\mathbf{1}\{T_{ijk\ell}=t_1\}\mid \mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell},\mathbf{T}_{i,-j,k}=\mathbf{t}_{i,-j,k}]\Pr{}^{\mathrm{R}}(\mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell},\mathbf{T}_{i,-j,k}=\mathbf{t}_{i,-j,k})\}$$

$$= \sum_{(\mathbf{t}_{ijk,-\ell},\mathbf{t}_{i,-j,k})}\Pr{}^{*}(\mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell},\mathbf{T}_{i,-j,k}=\mathbf{t}_{i,-j,k})$$

$$= 1$$

where the first equality follows from the rule of conditional expectation, the second from the fact that weights are fixed within the second expectation, the third from the definition of probability and weights, and the final equality also from the definition of probability.

Therefore, we obtain,

$$\frac{1}{NJK}\sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{k=1}^{K}\mathbf{1}\{T_{ijk\ell}=t_1\}w_{ijk\ell}\xrightarrow{p}1,$$

which completes the proof. □

## D.2 Consistency of Simple Difference-in-Means Under Marginal Population Randomization

Under the assumption of no three-way or higher-order interactions, the following simple difference-in-means is consistent for the pAMCE after randomizing profiles according to the marginal population randomization design (equation (3)).

$$\frac{\sum_{i=1}^N \sum_{j=1}^J \sum_{k=1}^K \mathbf{1}\{T_{ijk\ell} = t_1\} Y_{ijk}}{\sum_{i=1}^N \sum_{j=1}^J \sum_{k=1}^K \mathbf{1}\{T_{ijk\ell} = t_1\}} - \frac{\sum_{i=1}^N \sum_{j=1}^J \sum_{k=1}^K \mathbf{1}\{T_{ijk\ell} = t_0\} Y_{ijk}}{\sum_{i=1}^N \sum_{j=1}^J \sum_{k=1}^K \mathbf{1}\{T_{ijk\ell} = t_0\}} \xrightarrow{p} \tau_\ell^*(t_1, t_0)$$

**Proof.** Under the assumption of no three-way or higher-order interactions, the potential outcomes can be modeled as a function of all main terms and all two-way interactions between factors $(\mathbf{t}_{ijk}, \mathbf{t}_{i,-j,k})$ in the following fashion,

$$\begin{aligned} Y_{ik}(\mathbf{t}_{ijk}, \mathbf{t}_{i,-j,k}) = \quad & \tilde{\alpha}_{ik} + \sum_{j=1}^J \sum_{\ell=1}^L \mathbf{X}_{ijk\ell}^\top \tilde{\beta}_{ik,j\ell} \\ + \sum_{j=1}^J \sum_{\ell=1}^L \sum_{\ell' \neq \ell} (\mathbf{X}_{ijk\ell} \times \mathbf{X}_{ijk\ell'})^\top \tilde{\gamma}_{ik,j\ell\ell'} & + \sum_{j=1}^J \sum_{j' \neq j} \sum_{\ell=1}^L \sum_{\ell'=1}^L (\mathbf{X}_{ijk\ell} \times \mathbf{X}_{ij'k\ell'})^\top \tilde{\delta}_{ik,jj'\ell\ell'} + \tilde{\epsilon}_{ijk} \end{aligned}$$

where $\mathbf{X}_{ijk\ell}$ is a vector of $(D_\ell - 1)$ dummy variables for the levels of $t_{ijk\ell}$ excluding the baseline level, $\times$ represents the cartesian product operator, e.g., $(\mathbf{X}_{ijk\ell} \times \mathbf{X}_{ijk\ell'})^\top \tilde{\gamma}_{ik,j\ell\ell'} = \sum_{d=1}^{D_\ell - 1} \sum_{d'=1}^{D_{\ell'}-1} X_{ijk\ell d} X_{ijk\ell' d'} \tilde{\gamma}_{ik,j\ell d \ell' d'}$, and $\tilde{\epsilon}_{ijk}$ is the error term. Then,

$$\begin{aligned} & \sum_{(\mathbf{t}_{ijk,-\ell}, \mathbf{t}_{i,-j,k})} \{Y_{ijk}(t_1, \mathbf{t}_{ijk,-\ell}, \mathbf{t}_{i,-j,k}) - Y_{ijk}(t_0, \mathbf{t}_{ijk,-\ell}, \mathbf{t}_{i,-j,k})\} \Pr^*(\mathbf{T}_{ijk,-\ell} = \mathbf{t}_{ijk,-\ell}, \mathbf{T}_{i,-j,k} = \mathbf{t}_{i,-j,k}) \\ = & \sum_{(\mathbf{t}_{ijk,-\ell}, \mathbf{t}_{i,-j,k})} \{Y_{ijk}(t_1, \mathbf{t}_{ijk,-\ell}, \mathbf{t}_{i,-j,k}) - Y_{ijk}(t_0, \mathbf{t}_{ijk,-\ell}, \mathbf{t}_{i,-j,k})\} \prod_{\ell' \neq \ell} \Pr^*(\mathbf{T}_{ijk\ell'} = \mathbf{t}_{ijk\ell'}) \prod_{\ell''} \Pr^*(\mathbf{T}_{i,-j,k,\ell''} = \mathbf{t}_{i,-j,k,\ell''}) \end{aligned}$$

where the second expression only uses marginal distributions of each factor separately. Therefore, under the assumption of no three-way or higher-order interaction, the approximation of the joint distribution by the multiplication of each marginal distribution produces the same point estimate as the one based on the exact joint distribution.

Therefore, under the assumption of no three-way or higher-order interaction, weights are simplified as:

$$w_{ijk\ell} = \frac{1}{\Pr^{\mathsf{R}}(T_{ijk\ell} \mid \mathbf{T}_{ijk,-\ell}, \mathbf{T}_{i,-j,k})} \times \frac{\prod_{\ell' \neq \ell} \Pr^*(\mathbf{T}_{ijk\ell'} = \mathbf{t}_{ijk\ell'}) \prod_{\ell''} \Pr^*(\mathbf{T}_{i,-j,k,\ell''} = \mathbf{t}_{i,-j,k,\ell''})}{\Pr^{\mathsf{R}}(\mathbf{T}_{ijk,-\ell}, \mathbf{T}_{i,-j,k})}.$$

When the marginal population randomization design is used,

$$\begin{aligned} w_{ijk\ell}^{\mathtt{Mar}} &= \frac{1}{\Pr^*(T_{ijk\ell})} \times \frac{\prod_{\ell' \neq \ell} \Pr^*(\mathbf{T}_{ijk\ell'} = \mathbf{t}_{ijk\ell'}) \prod_{\ell''} \Pr^*(\mathbf{T}_{i,-j,k,\ell''} = \mathbf{t}_{i,-j,k,\ell''})}{\prod_{\ell' \neq \ell} \Pr^*(\mathbf{T}_{ijk\ell'} = \mathbf{t}_{ijk\ell'}) \prod_{\ell''} \Pr^*(\mathbf{T}_{i,-j,k,\ell''} = \mathbf{t}_{i,-j,k,\ell''})} \\ &= \frac{1}{\Pr^*(T_{ijk\ell})}. \end{aligned}$$

Therefore, the weighted difference-in-means becomes the simple difference-in-means.

$$\begin{aligned} & \frac{\sum_{i=1}^N \sum_{j=1}^J \sum_{k=1}^K \mathbf{1}\{T_{ijk\ell} = t_1\} w_{ijk\ell}^{\mathtt{Mar}} Y_{ijk}}{\sum_{i=1}^N \sum_{j=1}^J \sum_{k=1}^K \mathbf{1}\{T_{ijk\ell} = t_1\} w_{ijk\ell}^{\mathtt{Mar}}} - \frac{\sum_{i=1}^N \sum_{j=1}^J \sum_{k=1}^K \mathbf{1}\{T_{ijk\ell} = t_0\} w_{ijk\ell}^{\mathtt{Mar}} Y_{ijk}}{\sum_{i=1}^N \sum_{j=1}^J \sum_{k=1}^K \mathbf{1}\{T_{ijk\ell} = t_0\} w_{ijk\ell}^{\mathtt{Mar}}} \\ = & \frac{\sum_{i=1}^N \sum_{j=1}^J \sum_{k=1}^K \mathbf{1}\{T_{ijk\ell} = t_1\} Y_{ijk}}{\sum_{i=1}^N \sum_{j=1}^J \sum_{k=1}^K \mathbf{1}\{T_{ijk\ell} = t_1\}} - \frac{\sum_{i=1}^N \sum_{j=1}^J \sum_{k=1}^K \mathbf{1}\{T_{ijk\ell} = t_0\} Y_{ijk}}{\sum_{i=1}^N \sum_{j=1}^J \sum_{k=1}^K \mathbf{1}\{T_{ijk\ell} = t_0\}}. \end{aligned}$$

Based on Theorem 1,

$$\frac{\sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{k=1}^{K}\mathbf{1}\{T_{ijk\ell}=t_1\}Y_{ijk}}{\sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{k=1}^{K}\mathbf{1}\{T_{ijk\ell}=t_1\}} - \frac{\sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{k=1}^{K}\mathbf{1}\{T_{ijk\ell}=t_0\}Y_{ijk}}{\sum_{i=1}^{N}\sum_{j=1}^{J}\sum_{k=1}^{K}\mathbf{1}\{T_{ijk\ell}=t_0\}} \xrightarrow{p} \tau_\ell^*(t_1,t_0),$$

which completes the proof. □

## D.3 Optimality of the Mixed Randomization Design

Here, we investigate the Neyman variance of the following inverse probability weighting estimator that corresponds to the weighted difference-in-means estimator (equation (5)).

$$\widehat{\tau}_\ell^{\texttt{IPW}}(t_1,t_0) \;=\; \frac{1}{NK}\sum_{i=1}^{N}\sum_{k=1}^{K}\mathbf{1}\{T_{ijk\ell}=t_1\}w_{ijk\ell}Y_{ijk} - \frac{1}{NK}\sum_{i=1}^{N}\sum_{k=1}^{K}\mathbf{1}\{T_{ijk\ell}=t_0\}w_{ijk\ell}Y_{ijk}, \tag{A2}$$

We show that the mixed randomization design minimizes the Neyman variance when there is a single main factor and the assumption of no cross-profile interactions holds.

**Proof.** We can write the variance of the estimator as

$$\begin{aligned}
&\mathrm{Var}(\tau_\ell^{\texttt{IPW}}(t_1;t_0))\\
&= \mathrm{Var}\left(\frac{1}{NK}\sum_{i,k}\mathbf{1}\{T_{ijk\ell}=t_1\}w_{ijk\ell}Y_{ijk}\right) + \mathrm{Var}\left(\frac{1}{NK}\sum_{i,k}\mathbf{1}\{T_{ijk\ell}=t_0\}w_{ijk\ell}Y_{ijk}\right)\\
&\quad - 2\mathrm{Cov}\left(\frac{1}{NK}\sum_{i,k}\mathbf{1}\{T_{ijk\ell}=t_1\}w_{ijk\ell}Y_{ijk}, \frac{1}{NK}\sum_{i,k}\mathbf{1}\{T_{ijk\ell}=t_0\}w_{ijk\ell}Y_{ijk}\right).
\end{aligned}$$

First, we focus on the first term.

$$\mathrm{Var}\left(\frac{1}{NK}\sum_{i,k}\mathbf{1}\{T_{ijk\ell}=t_1\}w_{ijk\ell}Y_{ijk}\right) = \frac{1}{N^2K^2}\sum_{i,k}\mathrm{Var}\left(\mathbf{1}\{T_{ijk\ell}=t_1\}w_{ijk\ell}Y_{ijk}\right),$$

because treatments are independently randomized across individuals $i$ and task positions $k$. Focusing on the expression inside the summation,

$$\begin{aligned}
&\mathrm{Var}\left(\mathbf{1}\{T_{ijk\ell}=t_1\}w_{ijk\ell}Y_{ijk}\right)\\
&= \mathrm{Var}\Bigg(\sum_{\substack{\mathbf{t}_{ijk,-\ell},\\ \mathbf{t}_{i,-j,k}}}\mathbf{1}\{T_{ijk\ell}=t_1,\mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell},\mathbf{T}_{i,-j,k}=\mathbf{t}_{i,-j,k}\}Y_{ijk}(t_1,\mathbf{t}_{ijk,-\ell},\mathbf{t}_{i,-j,k})\\
&\qquad\qquad \times \frac{\Pr^*(\mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell},\mathbf{T}_{i,-j,k}=\mathbf{t}_{i,-j,k})}{\Pr^{\mathrm{R}}(T_{ijk\ell}=t_1,\mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell},\mathbf{T}_{i,-j,k}=\mathbf{t}_{i,-j,k})}\Bigg)\\
&= \sum_{\substack{\mathbf{t}_{ijk,-\ell},\\ \mathbf{t}_{i,-j,k}}} Y_{ijk}(t_1,\mathbf{t}_{ijk,-\ell},\mathbf{t}_{i,-j,k})^2\frac{\Pr^*(\mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell},\mathbf{T}_{i,-j,k}=\mathbf{t}_{i,-j,k})^2}{\Pr^{\mathrm{R}}(T_{ijk\ell}=t_1,\mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell},\mathbf{T}_{i,-j,k}=\mathbf{t}_{i,-j,k})^2}\\
&\qquad \times \mathrm{Var}\left(\mathbf{1}\{T_{ijk\ell}=t_1,\mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell},\mathbf{T}_{i,-j,k}=\mathbf{t}_{i,-j,k}\}\right)\\
&+ \sum_{\substack{\mathbf{t}_{ijk,-\ell},\\ \mathbf{t}_{i,-j,k}}}\sum_{\substack{\mathbf{t}'_{ijk,-\ell},\\ \mathbf{t}'_{i,-j,k}}} Y_{ijk}(t_1,\mathbf{t}_{ijk,-\ell},\mathbf{t}_{i,-j,k})Y_{ijk}(t_1,\mathbf{t}'_{ijk,-\ell},\mathbf{t}'_{i,-j,k})\\
&\qquad \times \frac{\Pr^*(\mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell},\mathbf{T}_{i,-j,k}=\mathbf{t}_{i,-j,k})}{\Pr^{\mathrm{R}}(T_{ijk\ell}=t_1,\mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell},\mathbf{T}_{i,-j,k}=\mathbf{t}_{i,-j,k})}\frac{\Pr^*(\mathbf{T}_{ijk,-\ell}=\mathbf{t}'_{ijk,-\ell},\mathbf{T}_{i,-j,k}=\mathbf{t}'_{i,-j,k})}{\Pr^{\mathrm{R}}(T_{ijk\ell}=t_1,\mathbf{T}_{ijk,-\ell}=\mathbf{t}'_{ijk,-\ell},\mathbf{T}_{i,-j,k}=\mathbf{t}'_{i,-j,k})}\\
&\qquad \times \mathrm{Cov}\left(\mathbf{1}\{T_{ijk\ell}=t_1,\mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell},\mathbf{T}_{i,-j,k}=\mathbf{t}_{i,-j,k}\},\mathbf{1}\{T_{ijk\ell}=t_1,\mathbf{T}_{ijk,-\ell}=\mathbf{t}'_{ijk,-\ell},\mathbf{T}_{i,-j,k}=\mathbf{t}'_{i,-j,k}\}\right)
\end{aligned}$$

$$= \sum_{\substack{\mathbf{t}_{ijk,-\ell},\\ \mathbf{t}_{i,-j,k}}} Y_{ijk}(t_1, \mathbf{t}_{ijk,-\ell}, \mathbf{t}_{i,-j,k})^2 \frac{\Pr^*(\mathbf{T}_{ijk,-\ell} = \mathbf{t}_{ijk,-\ell}, \mathbf{T}_{i,-j,k} = \mathbf{t}_{i,-j,k})^2}{\Pr^{\mathtt{R}}(T_{ijk\ell} = t_1, \mathbf{T}_{ijk,-\ell} = \mathbf{t}_{ijk,-\ell}, \mathbf{T}_{i,-j,k} = \mathbf{t}_{i,-j,k})^2}$$

$$\times \Pr^{\mathtt{R}}\Big(T_{ijk\ell} = t_1, \mathbf{T}_{ijk,-\ell} = \mathbf{t}_{ijk,-\ell}, \mathbf{T}_{i,-j,k} = \mathbf{t}_{i,-j,k}\Big) \times \Big\{1 - \Pr^{\mathtt{R}}\Big(T_{ijk\ell} = t_1, \mathbf{T}_{ijk,-\ell} = \mathbf{t}_{ijk,-\ell}, \mathbf{T}_{i,-j,k} = \mathbf{t}_{i,-j,k}\Big)\Big\}$$

$$- \sum_{\substack{\mathbf{t}_{ijk,-\ell},\\ \mathbf{t}_{i,-j,k}}} \sum_{\substack{\mathbf{t}'_{ijk,-\ell},\\ \mathbf{t}'_{i,-j,k}}} Y_{ijk}(t_1, \mathbf{t}_{ijk,-\ell}, \mathbf{t}_{i,-j,k}) Y_{ijk}(t_1, \mathbf{t}'_{ijk,-\ell}, \mathbf{t}'_{i,-j,k})$$

$$\times \Pr^*(\mathbf{T}_{ijk,-\ell} = \mathbf{t}_{ijk,-\ell}, \mathbf{T}_{i,-j,k} = \mathbf{t}_{i,-j,k}) \Pr^*(\mathbf{T}_{ijk,-\ell} = \mathbf{t}'_{ijk,-\ell}, \mathbf{T}_{i,-j,k} = \mathbf{t}'_{i,-j,k}),$$

where the first equality follow from the definition of potential outcomes and weights we propose, the second from the definition of variance, the third from the definition of Bernoulli distribution. Therefore,

$$\mathrm{Var}\Big(\frac{1}{NK} \sum_{i,k}^{n} \mathbf{1}\{T_{ijk\ell} = t_1\} w_{ijk\ell} Y_{ijk}\Big)$$

$$= \frac{1}{N^2K^2} \sum_{i,k} \sum_{\substack{\mathbf{t}_{ijk,-\ell},\\ \mathbf{t}_{i,-j,k}}} Y_{ijk}(t_1, \mathbf{t}_{ijk,-\ell}, \mathbf{t}_{i,-j,k})^2 \frac{\Pr^*(\mathbf{T}_{ijk,-\ell} = \mathbf{t}_{ijk,-\ell}, \mathbf{T}_{i,-j,k} = \mathbf{t}_{i,-j,k})^2}{\Pr^{\mathtt{R}}(T_{ijk\ell} = t_1, \mathbf{T}_{ijk,-\ell} = \mathbf{t}_{ijk,-\ell}, \mathbf{T}_{i,-j,k} = \mathbf{t}_{i,-j,k})}$$

$$\times \Big\{1 - \Pr^{\mathtt{R}}\Big(T_{ijk\ell} = t_1, \mathbf{T}_{ijk,-\ell} = \mathbf{t}_{ijk,-\ell}, \mathbf{T}_{i,-j,k} = \mathbf{t}_{i,-j,k}\Big)\Big\}$$

$$- \sum_{\substack{\mathbf{t}_{ijk,-\ell},\\ \mathbf{t}_{i,-j,k}}} \sum_{\substack{\mathbf{t}'_{ijk,-\ell},\\ \mathbf{t}'_{i,-j,k}}} Y_{ijk}(t_1, \mathbf{t}_{ijk,-\ell}, \mathbf{t}_{i,-j,k}) Y_{ijk}(t_1, \mathbf{t}'_{ijk,-\ell}, \mathbf{t}'_{i,-j,k})$$

$$\times \Pr^*(\mathbf{T}_{ijk,-\ell} = \mathbf{t}_{ijk,-\ell}, \mathbf{T}_{i,-j,k} = \mathbf{t}_{i,-j,k}) \Pr^*(\mathbf{T}_{ijk,-\ell} = \mathbf{t}'_{ijk,-\ell}, \mathbf{T}_{i,-j,k} = \mathbf{t}'_{i,-j,k}),$$

where the second term does not contain expressions related to $\Pr^{\mathtt{R}}()$ and hence it is the same for any randomized design.

Next, we focus on the third term of the variance.

$$\mathrm{Cov}\left(\frac{1}{NK} \sum_{i,k} \mathbf{1}\{T_{ijk\ell} = t_1\} w_{ijk\ell} Y_{ijk}, \frac{1}{NK} \sum_{i,k} \mathbf{1}\{T_{ijk\ell} = t_0\} w_{ijk\ell} Y_{ijk}\right)$$

$$= \frac{1}{N^2K^2} \sum_{i,k} \mathrm{Cov}\Big(\mathbf{1}\{T_{ijk\ell} = t_1\} w_{ijk\ell} Y_{ijk}, \mathbf{1}\{T_{ijk\ell} = t_0\} w_{ijk\ell} Y_{ijk}\Big)$$

$$= -\frac{1}{N^2K^2} \sum_{i,k} \Bigg\{ \sum_{\substack{\mathbf{t}_{ijk,-\ell},\\ \mathbf{t}_{i,-j,k}}} \{Y_{ijk}(T_{ijk\ell} = t_1, \mathbf{T}_{ijk,-\ell} = \mathbf{t}_{ijk,-\ell}, \mathbf{T}_{i,-j,k} = \mathbf{t}_{i,-j,k}) \Pr^*(\mathbf{T}_{ijk,-\ell} = \mathbf{t}_{ijk,-\ell}, \mathbf{T}_{i,-j,k} = \mathbf{t}_{i,-j,k})\}$$

$$\times \sum_{\substack{\mathbf{t}_{ijk,-\ell},\\ \mathbf{t}_{i,-j,k}}} \{Y_{ijk}(T_{ijk\ell} = t_0, \mathbf{T}_{ijk,-\ell} = \mathbf{t}_{ijk,-\ell}, \mathbf{T}_{i,-j,k} = \mathbf{t}_{i,-j,k}) \Pr^*(\mathbf{T}_{ijk,-\ell} = \mathbf{t}_{ijk,-\ell}, \mathbf{T}_{i,-j,k} = \mathbf{t}_{i,-j,k})\} \Bigg\},$$

where the first equality comes from the fact that treatments are independently randomized across individuals $i$ and task positions $k$, and the second from the definition of covariance. Because all the expressions are not related to $\Pr^{\mathtt{R}}()$, this covariance is the same for any randomized designs.

We now solve the minimization problem of the Neyman variance with respect to $\Pr^{\mathtt{R}}()$. To compare alternative experimental designs, we average over the potential outcomes unknown to researchers.

$$\mathbb{E}_{Y(\mathbf{t})}\left[\mathrm{Var}\Big(\frac{1}{NK} \sum_{i,k}^{n} \mathbf{1}\{T_{ijk\ell} = t_1\} w_{ijk\ell} Y_{ijk}\Big)\right]$$

$$= \frac{1}{N^2K^2}\sum_{i,k}\sum_{\substack{\mathbf{t}_{ijk,-\ell},\\ \mathbf{t}_{i,-j,k}}} \mathbb{E}_{Y(\mathbf{t})}[Y_{ijk}(t_1,\mathbf{t}_{ijk,-\ell},\mathbf{t}_{i,-j,k})^2]\frac{\Pr^*(\mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell},\mathbf{T}_{i,-j,k}=\mathbf{t}_{i,-j,k})^2}{\Pr^{\mathrm{R}}(T_{ijk\ell}=t_1,\mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell},\mathbf{T}_{i,-j,k}=\mathbf{t}_{i,-j,k})}$$
$$\times\left\{1-\Pr^{\mathrm{R}}\left(T_{ijk\ell}=t_1,\mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell},\mathbf{T}_{i,-j,k}=\mathbf{t}_{i,-j,k}\right)\right\}$$
$$-\sum_{\substack{\mathbf{t}_{ijk,-\ell},\\ \mathbf{t}_{i,-j,k}}}\sum_{\substack{\mathbf{t}'_{ijk,-\ell},\\ \mathbf{t}'_{i,-j,k}}} \mathbb{E}_{Y(\mathbf{t})}[Y_{ijk}(t_1,\mathbf{t}_{ijk,-\ell},\mathbf{t}_{i,-j,k})Y_{ijk}(t_1,\mathbf{t}'_{ijk,-\ell},\mathbf{t}'_{i,-j,k})]$$
$$\times\Pr^*(\mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell},\mathbf{T}_{i,-j,k}=\mathbf{t}_{i,-j,k})\Pr^*(\mathbf{T}_{ijk,-\ell}=\mathbf{t}'_{ijk,-\ell},\mathbf{T}_{i,-j,k}=\mathbf{t}'_{i,-j,k}),$$

where $\mathbb{E}_{Y(\mathbf{t})}$ is the expectation over the uniform distribution of the potential outcomes table. Therefore, $\mathbb{E}_{Y(\mathbf{t})}[Y_{ijk}(t_1,\mathbf{t}_{ijk,-\ell},\mathbf{t}_{i,-j,k})^2]$ and $\mathbb{E}_{Y(\mathbf{t})}[Y_{ijk}(t_1,\mathbf{t}_{ijk,-\ell},\mathbf{t}_{i,-j,k})Y_{ijk}(t_1,\mathbf{t}'_{ijk,-\ell},\mathbf{t}'_{i,-j,k})]$ are both constants. In addition, to compare experimental designs, we can remove all the terms that don't have $\Pr^{\mathrm{R}}()$. Taken together, we can focus on the following minimization problem under the assumption of no cross-profile interactions.

$$\min_{\Pr^{\mathrm{R}}()}\sum_{d=0}^{D_\ell-1}\sum_{\mathbf{t}_{ijk,-\ell}}\frac{\Pr^*(\mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell})^2}{\Pr^{\mathrm{R}}(T_{ijk\ell}=t_d,\mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell})}\quad \text{s.t.}\quad \sum_{d=0}^{D_\ell-1}\sum_{\mathbf{t}_{ijk,-\ell}}\Pr^{\mathrm{R}}(T_{ijk\ell}=t_d,\mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell})=1.$$

Then, by using the Lagrange multiplier, we can solve:

$$\min_{\Pr^{\mathrm{R}}()} L$$
$$\text{where}\quad L=\sum_{d=0}^{D_\ell-1}\sum_{\mathbf{t}_{ijk,-\ell}}\frac{\Pr^*(\mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell})^2}{\Pr^{\mathrm{R}}(T_{ijk\ell}=t_d,\mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell})}+\lambda\left(\sum_{d=0}^{D_\ell-1}\sum_{\mathbf{t}_{ijk,-\ell}}\Pr^{\mathrm{R}}(T_{ijk\ell}=t_d,\mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell})-1\right)$$
$$\text{and}\quad \lambda>0.$$

Therefore,

$$\frac{\partial L}{\partial \Pr^{\mathrm{R}}(T_{ijk\ell}=t_d,\mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell})}=0\iff \Pr^{\mathrm{R}}(T_{ijk\ell}=t_d,\mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell})=\frac{1}{\sqrt{\lambda}}\Pr^*(\mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell}).$$

In addition,

$$\sum_{d=0}^{D_\ell-1}\sum_{\mathbf{t}_{ijk,-\ell}}\Pr^{\mathrm{R}}(T_{ijk\ell}=t_d,\mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell})=1\iff\sqrt{\lambda}=D_\ell.$$

Hence, the optimal randomization distribution is the mixed randomization design.

$$\Pr^{\mathrm{R}}(T_{ijk\ell}=t_d,\mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell})=\frac{1}{D_\ell}\Pr^*(\mathbf{T}_{ijk,-\ell}=\mathbf{t}_{ijk,-\ell}),$$

which completes the proof. □

# E Diagnostic Tools for Model-based Analysis

Here, we introduce a set of diagnostic tools that are designed to help researchers assess the validity of modeling assumptions. These tools are essential for a successful implementation of the proposed model-based exploratory analysis.

**Specification test.** We first introduce a specification test that assesses the validity of all the modeling assumptions as a whole under the uniform randomization design. The idea is that if the modeling assumptions were violated, the estimated uAMCE using the model-based approach would differ from the simple difference-in-means estimator, which is unbiased under the uniform randomization design. We test whether the difference in the two estimates are statistically distinguishable from zero using bootstrap.

If the model-based estimate of the uAMCE is significantly different from the difference-in-means estimate, at least one modeling assumption is likely to be violated. Because we rely on the same modeling assumptions when estimating the pAMCE, it is likely that a model-based estimate of the pAMCE is also biased. This diagnostic tool can be implemented with or without regularization. We caution that rejecting the null hypothesis of no difference does not tell us which modeling assumption is violated — the absence of higher-order interaction or the absence of strong regularization bias.

**Regularization bias.** The proposed regularization procedure given in Section 4.2.3 uses cross-fitting to minimize the possible bias due to incorrectly shrinking coefficients to zero. In practice, however, one should check how regularization affects the estimate of the pAMCE. We suggest examining the bootstrap distribution of the estimated pAMCE separately for each factor. When a regularization bias is substantial, the bootstrap distribution often differs significantly from the normal distribution.

# F Simulation Studies

In this section, we conduct simulation studies to evaluate the performance of the proposed methodology. Specifically, we examine the following three aspects: the relative efficiency of the mixed randomization design over the uniform and marginal population randomization designs, the bias-variance tradeoff of the regularization approach, and the advantage of the design-based confirmatory analysis over the model-based exploratory analysis.

## F.1 The Setup

To make simulation settings realistic, we utilize the data from the conjoint analysis about attitudes toward immigrants (Hainmueller *et al.*, 2015). The original study has the following seven factors where the number of levels is shown in parentheses; `Age` (4), `Education` (3), `Gender` (2), `Integration` (4), `Language` (4), `Origin` (8), `Year since arrival` (4). To construct the population distribution of immigrant profiles, we follow Hainmueller *et al.* (2015) and use the information from the actual referendums conducted in Switzerland, giving us the marginal distribution of each factor (see Table A3). Finally, we use the following linear utility model as the true data generating process,

$$\begin{aligned}
\widetilde{Y}_{ijk} = \quad & 0.1 + \mathbf{X}_{ijk,\texttt{Gen}}^\top \widetilde{\beta}_{\texttt{Gen}} + \mathbf{X}_{ijk,\texttt{Ori}}^\top \widetilde{\beta}_{\texttt{Ori}} + \mathbf{X}_{ijk,\texttt{Age}}^\top \widetilde{\beta}_{\texttt{Age}} + \mathbf{X}_{ijk,\texttt{Year}}^\top \widetilde{\beta}_{\texttt{Year}} + \mathbf{X}_{ijk,\texttt{Edu}}^\top \widetilde{\beta}_{\texttt{Edu}} + \mathbf{X}_{ijk,\texttt{Int}}^\top \widetilde{\beta}_{\texttt{Int}} \\
& + \mathbf{X}_{ijk,\texttt{Lan}}^\top \widetilde{\beta}_{\texttt{Lan}} + (\mathbf{X}_{ijk,\texttt{Year}} \bigtimes \mathbf{X}_{ijk,\texttt{Edu}})^\top \widetilde{\gamma}_{\texttt{Year,Edu}} + (\mathbf{X}_{ijk,\texttt{Edu}} \bigtimes \mathbf{X}_{ijk,\texttt{Lan}})^\top \widetilde{\gamma}_{\texttt{Edu,Lan}} \\
& + (\mathbf{X}_{ijk,\texttt{Edu}} \bigtimes \mathbf{X}_{ij'k,\texttt{Edu}})^\top \widetilde{\delta}_{\texttt{Edu,Edu}}, \\
\Pr(Y_{ijk} = 1 \mid \mathbf{X}_{ijk}, \mathbf{X}_{ij'k}) \ = \ & \left(\widetilde{Y}_{ijk} - \widetilde{Y}_{ij'k}\right) + 0.5,
\end{aligned}$$

| Factors | Levels |
|---|---|
| `Gender` | Male (0.69), Female (0.31) |
| `Origin` | Netherlands (0.01), Germany (0.18), Austria (0.04), Italy (0.21), Turkey (0.21), Croatia (0.05), Former Yugoslavia (0.23), Bosnia-Herzegovina (0.07) |
| `Age` | 21 years (0.30), 30 years (0.21), 41 years (0.33), 55 years (0.16) |
| `Years since arrival` | 14 years (0.26), 20 years (0.30), 29 years (0.14), Born in CH (0.30) |
| `Education` | Primary school (0.31), High school (0.60), University (0.09) |
| `Language` | Adequate (0.03), Good (0.09), Perfect (0.88) |
| `Integration` | Assimilated (0.37), Integrated (0.33), Indistinguishable (0.08), Familiar with Swiss traditions (0.22) |

Table A3: Factors, Levels, and Each Probability Used in Hainmueller *et al.* (2015). The factors were constructed in order to match the categories of the leaflets on which actual immigrants' characteristics were printed.

We choose the values of the coefficients such that there are substantial interaction effects, making the comparison of different methods clear.[10]

We compare four approaches, each of which corresponds to a different combination of an experimental design and its corresponding estimator; (1) Mixed Design: the mixed randomization design (equation (4)) and its corresponding weighted difference-in-means estimator (equation (5)), (2) Population Design: the marginal population randomization design (equation (2)) and its corresponding difference-in-means estimator (equation (7)), (3) Reg-regression: the uniform randomization and the regularized regression estimator (equation (14)), and (4) Regression: the uniform randomization and the non-regularized regression estimator (equation (11)). For Mixed Design, we specify one main factor of interest. For each simulation, the results reported here average over the results for each of the seven factors. Using a total of 1000 Monte Carlo simulations, we compute the bias, standard error, and root mean of squared error (RMSE) of each estimator as well as the coverage of 95% confidence intervals. We let the sample size vary from 1000 to 8000, i.e., $\{1000, 2000, 4000, 6000, 8000\}$.

## F.2 The Results

Figure A4 presents the results. First, as we expect from Theorem 1, both Mixed Design and Population Design induce little bias (see the upper left plot). The correctly specified Regression also suffers from little bias, whereas Reg-Regression has also little bias due to its flexible two-way interaction model. Second, in terms of statistical efficiency, because Mixed

[10]Specifically, we set $\widetilde{\beta}_{\texttt{Gen}} = -0.01$, $\widetilde{\beta}_{\texttt{Ori}} = (0, 0, 0, 0, 0, -0.002, -0.002)$, $\widetilde{\beta}_{\texttt{Age}} = (-0.005, -0.01, -0.01)$, $\widetilde{\beta}_{\texttt{Year}} = (0, 0.01, 0.01)$, $\widetilde{\beta}_{\texttt{Edu}} = (0.005, 0.02)$, $\widetilde{\beta}_{\texttt{Int}} = (0, -0.01, 0.01)$, $\widetilde{\beta}_{\texttt{Lan}} = (0.005, 0.01)$, $\widetilde{\gamma}_{\texttt{Year,Edu}} = (0.005, 0.005, 0.005, 0.01, 0.01, 0.01)$, $\widetilde{\gamma}_{\texttt{Edu,Lan}} = (0.005, 0.005, 0.01, 0.01)$, and $\widetilde{\delta}_{\texttt{Edu,Edu}} = (-0.0024, -0.0048, -0.0024, -0.0048)$.

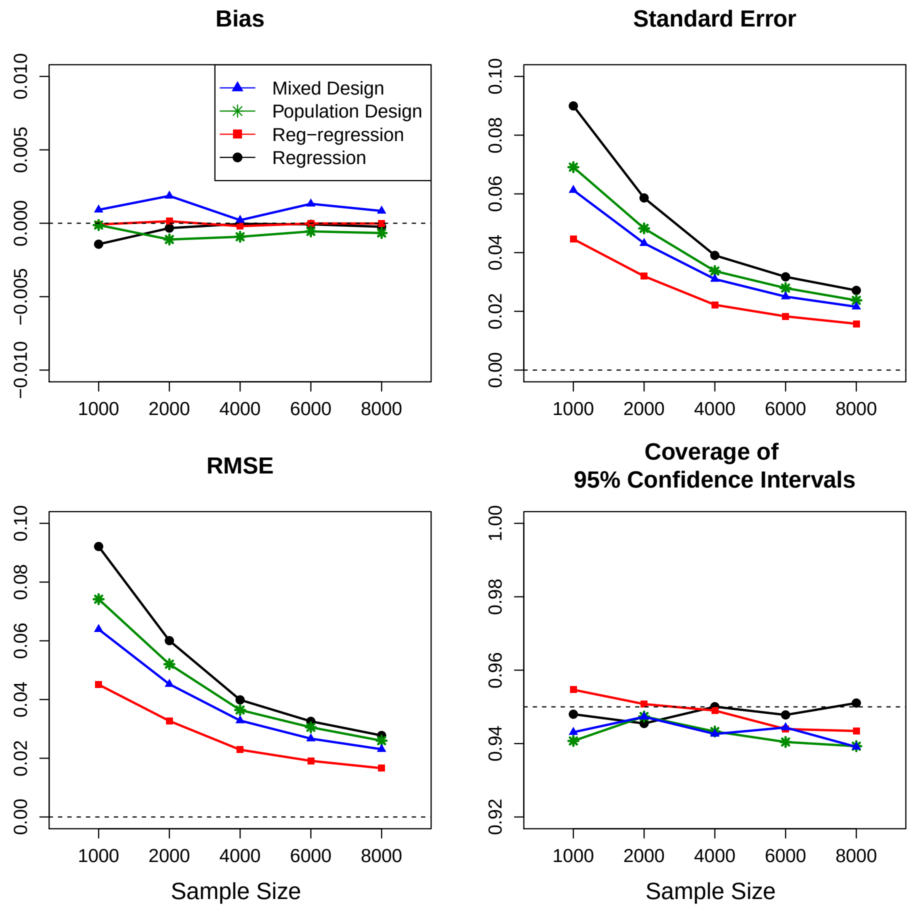



Figure A4: Comparison of Four Approaches in terms of Bias, Standard Error, RMSE, and the Coverage of 95% Confidence Intervals. We evaluate (1) the mixed randomization design and its corresponding weighted difference-in-means estimator (Mixed Design, blue square), (2) the joint population randomization design and its corresponding simple difference-in-means estimator (Population Design, green diamond), (3) the uniform randomization design and the regularized regression estimator (Reg-regression, red star), and (4) the uniform randomization design and the non-regularized regression estimator (Regression, black circle).

Design focuses on only one factor at a time, it has smaller standard errors than Population Design (see the upper right plot). Comparing the two model-based estimators, Reg-regression has smaller standard errors than Regression. The efficiency gain of Reg-regression is achieved by collapsing indistinguishable levels. In fact, this simulation shows that Reg-regression can achieve standard errors even smaller than the design-based confirmatory analysis when there are a lot of redundant levels. However, in some applications like Ono and Burden (2019), the design-based confirmatory analysis is more efficient. Whenever possible, we recommend the design-based confirmatory analysis because researchers can always implement the regularized approach after data collection if necessary. Finally, the coverage of the 95% confidence intervals is reasonable for all estimators.