Supplementary Materials

S1. Proofs

We collect notation from the main text and introduce new notation:

$$m_{\boldsymbol{x}}(y,z) = \mathbb{E}(Y_{i2} \mid D_i = 1, Z_i = z, Y_{i1} = y, \boldsymbol{X}_i = \boldsymbol{x}), \quad q_{\boldsymbol{x}}(y,z) = \Pr(Y_{i1} = y \mid D_i = 1, Z_i = z, \boldsymbol{X}_i = \boldsymbol{x}),$$

$$m^*_{\boldsymbol{x}}(y,z) = \mathbb{E}(Y_{i2} \mid D_i = 1, Z_i = z, Y^*_{i1} = y, \boldsymbol{X}_i = \boldsymbol{x}), \quad q^*_{\boldsymbol{x}}(y,z) = \Pr(Y^*_{i1} = y \mid D_i = 1, Z_i = z, \boldsymbol{X}_i = \boldsymbol{x}),$$

$$p_{\boldsymbol{x}}(y) = \Pr(Y_{i1} = y \mid Y^*_{i1} = y, G_i = c, \boldsymbol{X}_i = \boldsymbol{x}), \quad \xi_{\boldsymbol{x}}(y,z) = \mathrm{pr}(Y^*_{i1} = 1 \mid Z_i = z, D_i = 1, Y_{i1} = y, \boldsymbol{X}_i = \boldsymbol{x}),$$

$$r_{\boldsymbol{x}}(z) = \frac{q_{\boldsymbol{x}}(1, 2-z)\{1 - q_{\boldsymbol{x}}(1, 2-z)\}}{\{p_{\boldsymbol{x}}(1) - q_{\boldsymbol{x}}(1, 2-z)\} \cdot [q_{\boldsymbol{x}}(1, 2-z) - \{1 - p_{\boldsymbol{x}}(0)\}]}.$$

We will prove our results under the following weaker version of Assumption 1.

Assumption S1 (Strong Ignorability of Treatment Assignment)**.**

$$\begin{aligned} &Z_i \perp\!\!\!\perp D_i(z) \mid \boldsymbol{X}_i, \\ &Z_i \perp\!\!\!\perp \{Y^*_{i1}(z), Y_{i2}(z, y^*_1)\} \mid G_i = c, \boldsymbol{X}_i, \\ &0 < \Pr(Z_i = z \mid \boldsymbol{X}_i) < 1 \end{aligned}$$

for $z = 0, 1, 2$ and $y^*_1 = 0, 1$.

S1.1. Proof of Theorem 1

We first consider the average spillover effect, by the law of total probability,

$$\begin{aligned} \theta &= \sum_{\boldsymbol{x}} \{\mathbb{E}(Y_{i2}(1) \mid G_i = c, \boldsymbol{X}_i = \boldsymbol{x})\mathrm{pr}(\boldsymbol{X}_i = \boldsymbol{x} \mid G_i = c) - \mathbb{E}(Y_{i2}(0) \mid G_i = c, \boldsymbol{X}_i = \boldsymbol{x})\mathrm{pr}(\boldsymbol{X}_i = \boldsymbol{x} \mid G_i = c)\} \\ &= \sum_{\boldsymbol{x}} \{\mathbb{E}(Y_{i2} \mid Z_i = 1, G_i = c, \boldsymbol{X}_i = \boldsymbol{x}) - \mathbb{E}(Y_{i2}(2) \mid G_i = c, \boldsymbol{X}_i = \boldsymbol{x})\}\, \mathrm{pr}(\boldsymbol{X}_i = \boldsymbol{x} \mid G_i = c) \\ &= \sum_{\boldsymbol{x}} \{\mathbb{E}(Y_{i2} \mid Z_i = 1, D_i = 1, \boldsymbol{X}_i = \boldsymbol{x}) - \mathbb{E}(Y_{i2} \mid Z_i = 2, D_i = 1, \boldsymbol{X}_i = \boldsymbol{x})\}\, \mathrm{pr}(\boldsymbol{X}_i = \boldsymbol{x} \mid G_i = c), \qquad \text{(S1)} \end{aligned}$$

where the second inequality follows from Assumption 3. From Assumption S1,

$$
\begin{aligned}
\mathrm{pr}(\boldsymbol{X}_i = \boldsymbol{x} \mid G_i = c) &= \frac{\mathrm{pr}(G_i = c \mid \boldsymbol{X}_i = \boldsymbol{x})\mathrm{pr}(\boldsymbol{X}_i = \boldsymbol{x})}{\mathrm{pr}(G_i = c)} \\
&= \frac{\mathrm{pr}(G_i = c \mid \boldsymbol{X}_i = \boldsymbol{x})\mathrm{pr}(\boldsymbol{X}_i = \boldsymbol{x})}{\sum_{\boldsymbol{x}} \mathrm{pr}(G_i = c \mid \boldsymbol{X}_i = \boldsymbol{x})\mathrm{pr}(\boldsymbol{X}_i = \boldsymbol{x})} \\
&= \frac{\mathrm{pr}(D_i = 1 \mid Z_i \neq 0, \boldsymbol{X}_i = \boldsymbol{x})\mathrm{pr}(\boldsymbol{X}_i = \boldsymbol{x})}{\sum_{\boldsymbol{x}} \mathrm{pr}(D_i = 1 \mid Z_i \neq 0, \boldsymbol{X}_i = \boldsymbol{x})\mathrm{pr}(\boldsymbol{X}_i = \boldsymbol{x})}, \qquad \text{(S2)}
\end{aligned}
$$

which simplifies to $\mathrm{pr}(\boldsymbol{X}_i = \boldsymbol{x} \mid D_i = 1)$ if Assumption 1 holds. Plugging (S2) into (S1), we can obtain the identification formula for θ, which becomes the same as the one in Theorem 1 if Assumption 1 holds.

We then consider the average contagion and direct effects. We only need to identify $\mathbb{E}\{Y_{i2}(z, Y_{i1}^*(z')) \mid G_i = c, \boldsymbol{X}_i = \boldsymbol{x}\}$ for all $\boldsymbol{x}$ and $z, z' = 0, 1$. By the law of total probability, we have

$$
\begin{aligned}
& \mathbb{E}\{Y_{i2}(z, Y_{i1}^*(z')) \mid G_i = c, \boldsymbol{X}_i = \boldsymbol{x}\} \\
= & \sum_{y_1^*=0}^{1} \mathbb{E}\{Y_{i2}(z, y_{i1}^*) \mid G_i = c, Y_{i1}^*(z') = y_1^*, \boldsymbol{X}_i = \boldsymbol{x}\} \Pr\{Y_{i1}^*(z') = y_1^* \mid G_i = c, \boldsymbol{X}_i = \boldsymbol{x}\} \\
= & \sum_{y_1^*=0}^{1} \mathbb{E}\{Y_{i2}(z, y_1^*) \mid Z_i = z', G_i = c, Y_{i1}^*(z') = y_1^*, \boldsymbol{X}_i = \boldsymbol{x}\} \Pr(Y_{i1}^* = y_1^* \mid Z_i = z', G_i = c, \boldsymbol{X}_i = \boldsymbol{x}) \\
= & \sum_{y_1^*=0}^{1} \mathbb{E}\{Y_{i2}(z, y_1^*) \mid Z_i = z', G_i = c, \boldsymbol{X}_i = \boldsymbol{x}\} \Pr(Y_{i1}^* = y_1^* \mid Z_i = z', G_i = c, \boldsymbol{X}_i = \boldsymbol{x}) \\
= & \sum_{y_1^*=0}^{1} \mathbb{E}\{Y_{i2}(z, y_1^*) \mid Z_i = z, G_i = c, \boldsymbol{X}_i = \boldsymbol{x}\} \Pr(Y_{i1}^* = y_1^* \mid Z_i = z', G_i = c, \boldsymbol{X}_i = \boldsymbol{x}) \\
= & \sum_{y_1^*=0}^{1} \mathbb{E}\{Y_{i2}(z, y_1^*) \mid Z_i = z, G_i = c, Y_{i1}^*(z) = y_1^*, \boldsymbol{X}_i = \boldsymbol{x}\} \Pr(Y_{i1}^* = y_1^* \mid Z_i = z', G_i = c, \boldsymbol{X}_i = \boldsymbol{x}) \\
= & \sum_{y=0}^{1} m_{\boldsymbol{x}}^*(y, z) q_{\boldsymbol{x}}^*(y, z'), \qquad \text{(S3)}
\end{aligned}
$$

where the second and the fourth equalities follow from Assumption 1, and the third and the fifth equalities follow from Assumption 2. Therefore, we need only to identify $m_{\boldsymbol{x}}^*(y, z)$ and $q_{\boldsymbol{x}}^*(y, z')$ for $z, y = 0, 1$. From Assumption 4, we can identify $q_{\boldsymbol{x}}^*(y, 1) = q_{\boldsymbol{x}}(y, 1)$, and from Assumption 3, we can identify

$$q^*_{\boldsymbol{x}}(y,0) \quad = \quad \Pr(Y^*_{i1}=1 \mid Z_i=2, G_i=c, \boldsymbol{X}_i=\boldsymbol{x}) \;=\; q_{\boldsymbol{x}}(y,2).$$

Finally, Assumption 4 implies $m^*_{\boldsymbol{x}}(y,1)=m_{\boldsymbol{x}}(y,1)$, and by Assumptions 3 and 4, we have,

$$m^*_{\boldsymbol{x}}(y,0) \quad = \quad \mathbb{E}(Y_{i2} \mid Z_i=2, G_i=c, Y^*_{i1}=y^*_1, \boldsymbol{X}_i=\boldsymbol{x}) \;=\; m_{\boldsymbol{x}}(y,2).$$

By plugging the identification formulas of $q^*_{\boldsymbol{x}}(y,z)$ and $m^*_{\boldsymbol{x}}(y,z)$ for $z=0,1$ into equation (S3), we can obtain the identification formulas for the average contagion and direct effects. They become the same as the identification formulas in Theorem 1 if Assumption 1 holds. □

S1.2. Proof of Theorem 2

First, because the expression of θ does not include Y_{i1}, the identification formula does not change without Assumption 4, i.e.,

$$\theta \;=\; \sum_{\boldsymbol{x}} \Pr(\boldsymbol{X}_i=\boldsymbol{x} \mid G_i=c)\cdot\{m_{\boldsymbol{x}}(1,1)q_{\boldsymbol{x}}(1,1)+m_{\boldsymbol{x}}(0,1)q_{\boldsymbol{x}}(0,1)-m_{\boldsymbol{x}}(1,2)q_{\boldsymbol{x}}(1,2)-m_{\boldsymbol{x}}(0,2)q_{\boldsymbol{x}}(0,2)\}.$$

For $z=1,2$, by the law of total probability and Assumption 5,

$$q_{\boldsymbol{x}}(1,z)=p_{\boldsymbol{x}}(1)\cdot q^*_{\boldsymbol{x}}(1,z)+(1-p_{\boldsymbol{x}}(0))\cdot\{1-q^*_{\boldsymbol{x}}(1,z)\}.$$

We then have

$$q^*_{\boldsymbol{x}}(1,z)=\frac{q_{\boldsymbol{x}}(1,z)-(1-p_{\boldsymbol{x}}(0))}{p_{\boldsymbol{x}}(1)+p_{\boldsymbol{x}}(0)-1}.$$

Again, by the law of total probability and Assumption 5, we have

$$m_{\boldsymbol{x}}(1,z)=m^*_{\boldsymbol{x}}(1,z)\cdot\xi_{\boldsymbol{x}}(1,z)+m^*_{\boldsymbol{x}}(0,z)\cdot\{1-\xi_{\boldsymbol{x}}(1,z)\},$$
$$m_{\boldsymbol{x}}(0,z)=m^*_{\boldsymbol{x}}(1,z)\cdot\xi_{\boldsymbol{x}}(0,z)+m^*_{\boldsymbol{x}}(0,z)\cdot\{1-\xi_{\boldsymbol{x}}(0,z)\},$$

from which we obtain

$$
\begin{aligned}
m^*_{\boldsymbol{x}}(1,z) &= \frac{(1-\xi_{\boldsymbol{x}}(0,z))m_{\boldsymbol{x}}(1,z)-(1-\xi_{\boldsymbol{x}}(1,z))m_{\boldsymbol{x}}(0,z)}{\xi_{\boldsymbol{x}}(1,z)-\xi_{\boldsymbol{x}}(0,z)}, \\
m^*_{\boldsymbol{x}}(0,z) &= \frac{\xi_{\boldsymbol{x}}(1,z)m_{\boldsymbol{x}}(0,z)-\xi_{\boldsymbol{x}}(0,z)m_{\boldsymbol{x}}(1,z)}{\xi_{\boldsymbol{x}}(1,z)-\xi_{\boldsymbol{x}}(0,z)}.
\end{aligned}
$$

From Theorem 1, for $z=0,1$,

$$
\begin{aligned}
\tau(z) &= \sum_{\boldsymbol{x}}\{m^*_{\boldsymbol{x}}(1,2-z)-m^*_{\boldsymbol{x}}(0,2-z)\}\{q^*_{\boldsymbol{x}}(1,1)-q^*_{\boldsymbol{x}}(1,2)\}\Pr(\boldsymbol{X}_i=\boldsymbol{x}) \\
&= \sum_{\boldsymbol{x}}\frac{m_{\boldsymbol{x}}(1,2-z)-m_{\boldsymbol{x}}(0,2-z)}{\xi_{\boldsymbol{x}}(1,2-z)-\xi_{\boldsymbol{x}}(0,2-z)}\cdot\frac{q_{\boldsymbol{x}}(1,1)-q_{\boldsymbol{x}}(1,2)}{p_{\boldsymbol{x}}(1)+p_{\boldsymbol{x}}(0)-1}\cdot\Pr(\boldsymbol{X}_i=\boldsymbol{x}),
\end{aligned}
$$

whereas

$$
\begin{aligned}
&\xi_{\boldsymbol{x}}(1,2-z)-\xi_{\boldsymbol{x}}(0,2-z) = \frac{p_{\boldsymbol{x}}(1)\cdot q^*_{\boldsymbol{x}}(1,2-z)}{q_{\boldsymbol{x}}(1,2-z)}-\frac{(1-p_{\boldsymbol{x}}(1))\cdot q^*_{\boldsymbol{x}}(1,2-z)}{1-q_{\boldsymbol{x}}(1,2-z)} \\
=\ & \frac{\{p_{\boldsymbol{x}}(1)-q_{\boldsymbol{x}}(1,2-z)\}\cdot q^*_{\boldsymbol{x}}(1,2-z)}{q_{\boldsymbol{x}}(1,2-z)(1-q_{\boldsymbol{x}}(1,2-z))} \\
=\ & \frac{(p_{\boldsymbol{x}}(1)-q_{\boldsymbol{x}}(1,2-z))\cdot[q_{\boldsymbol{x}}(1,2-z)-\{1-p_{\boldsymbol{x}}(0)\}]}{q_{\boldsymbol{x}}(1,2-z)(1-q_{\boldsymbol{x}}(1,2-z))}\cdot\frac{1}{p_{\boldsymbol{x}}(1)+p_{\boldsymbol{x}}(0)-1}.
\end{aligned}
$$

Therefore, we have

$$
\tau(z)=\sum_{\boldsymbol{x}}[\Pr(\boldsymbol{X}_i=\boldsymbol{x}\mid G_i=c)\cdot r_{\boldsymbol{x}}(z)\{m_{\boldsymbol{x}}(1,2-z)-m_{\boldsymbol{x}}(0,2-z)\}\{q_{\boldsymbol{x}}(1,1)-q_{\boldsymbol{x}}(1,2)\}].
$$

For the average direct effect, we have

$$
\begin{aligned}
\eta(z) &= \theta-\tau(1-z) \\
&= \sum_{\boldsymbol{x}}\left[\Pr(\boldsymbol{X}_i=\boldsymbol{x}\mid G_i=c)\cdot\sum_{y=0}^{1}\Big(m_{\boldsymbol{x}}(y,1)q_{\boldsymbol{x}}(y,1)-m_{\boldsymbol{x}}(y,2)q_{\boldsymbol{x}}(y,2)\right. \\
&\qquad\left. -r_{\boldsymbol{x}}(1-z)m_{\boldsymbol{x}}(y,1+z)\{q_{\boldsymbol{x}}(y,1)-q_{\boldsymbol{x}}(y,2)\}\Big)\right] \\
&= \sum_{\boldsymbol{x}}\left[\Pr(\boldsymbol{X}_i=\boldsymbol{x}\mid G_i=c)\cdot\sum_{y=0}^{1}\Big(\{m_{\boldsymbol{x}}(y,1)-m_{\boldsymbol{x}}(y,2)\}q_{\boldsymbol{x}}(y,2-z)\right. \\
&\qquad\left. -\{1-r_{\boldsymbol{x}}(1-z)\}m_{\boldsymbol{x}}(y,1+z)\{q_{\boldsymbol{x}}(y,2)-q_{\boldsymbol{x}}(y,1)\}\Big)\right].
\end{aligned}
$$

Plugging (S2) into the above equations, we can obtain the identification formulas for the average contagion and direct effects. They become the same as those in Theorem 2 if Assumption 1 holds. □

S1.3. Proof of Corollary 1

We only need to drive the bounds for $r_{\boldsymbol{x}}(z)$. Because $p_{\boldsymbol{x}}(1) + p_{\boldsymbol{x}}(0) \geq p$ and $0 \leq p_{\boldsymbol{x}}(1), p_{\boldsymbol{x}}(0) \leq 1$, we can obtain the range of $(p_{\boldsymbol{x}}(1), 1 - p_{\boldsymbol{x}}(0))$ as,

$$1 - p_{\boldsymbol{x}}(0) \in [0, 2-p], \quad p_{\boldsymbol{x}}(1) \in [1 - p_{\boldsymbol{x}}(0) + p - 1, 1].$$

To obtain the bounds for $r_{\boldsymbol{x}}(z)$, we first look into the range of the following function:

$$f_a(x_1, x_2) = (a - x_1)(a - x_2), \quad a \in [0,1], \quad x_1 \in [0, 2-p], \quad x_2 \in [x_1 + p - 1, 1].$$

We enumerate all the cases for different relative magnitude among $2 - p$, $p - 1$ and a.

Case 1: $p \geq 3/2$

(a) When $a < 2 - p$, we have $x_2 < 1 - p \geq a$. $f_a(x_1, x_2)$ reaches its maximum $\{a - (2 - p)\}(a - 1)$ at $(x_1, x_2) = (2 - p, 1)$, and reaches its minimum $a(a - 1)$ at $(x_1, x_2) = (0, 1)$. Thus, we have $f_a(x_1, x_2) \in [a(a - 1), \{a - (2 - p)\})(a - 1)]$.

(b) When $a = 2 - p$, we have $x_2 < 1 - p \geq a$. $f_a(x_1, x_2)$ reaches its maximum 0 at $(x_1, x_2) = (2 - p, 1)$, and reaches its minimum $a(a - 1)$ at $(x_1, x_2) = (0, 1)$. Thus, we have $f_a(x_1, x_2) \in [a(a - 1), 0]$.

(c) When $2 - p < a < p - 1$, we have $x_2 < 2 - p < a < p - 1 \leq x_1$. $f_a(x_1, x_2)$ reaches its minimum $a(a - 1)$ at $(x_1, x_2) = (0, 1)$. If $a \leq 1/2$, then $f_a(x_1, x_2)$ reaches its maximum $\{a - (2 - p)\}(a - 1)$ at $(x_1, x_2) = (2 - p, 1)$ and if $a > 1/2$, then $f_a(x_1, x_2)$ reaches its maximum $\{a - (p - 1)\}a$ at $(x_1, x_2) = (0, p - 1)$.

(d) When $a = p - 1$, we have $x_1 \leq 2 - p \leq a$. $f_a(x_1, x_2)$ reaches its maximum 0 at $(x_1, x_2) = (0, p - 1)$, and reaches its minimum $a(a - 1)$ at $(x_1, x_2) = (0, 1)$. Thus, we have $f_a(x_1, x_2) \in [a(a - 1), 0]$.

(e) When $a > p-1$, we have $x_1 \leq 2-p \leq a$. $f_a(x_1,x_2)$ reaches its maximum $\{a-(p-1)\}a$ at $(x_1,x_2)=(0,p-1)$, and reaches its minimum $a(a-1)$ at $(x_1,x_2)=(0,1)$. Thus, we have $f_a(x_1,x_2) \in [a(a-1), \{a-(p-1)\}a]$.

Case 2: $p < 3/2$

(a) When $a \leq p-1$, we have $a \leq x_2$. $f_a(x_1,x_2)$ reaches its maximum $\{a-(2-p)\}(a-1)$ at $(x_1,x_2)=(2-p,1)$, and reaches its minimum $a(a-1)$ at $(x_1,x_2)=(0,1)$. Thus, we have $f_a(x_1,x_2) \in [a(a-1), \{a-(2-p)\})(a-1)]$.

(b) When $p-1 < a < 2-p$, $f_a(x_1,x_2)$ reaches its minimum $a(a-1)$ at $(x_1,x_2)=(0,1)$. If $a \leq 1/2$, then $f_a(x_1,x_2)$ reaches its maximum $\{a-(2-p)\}(a-1)$ at $(x_1,x_2)=(2-p,1)$ and if $a > 1/2$, then $f_a(x_1,x_2)$ reaches its maximum $\{a-(p-1)\}a$ at $(x_1,x_2)=(0,p-1)$.

(c) When $a \geq 2-p$, we have $a \geq x_1$. $f_a(x_1,x_2)$ reaches its maximum $\{a-(p-1)\}a$ at $(x_1,x_2)=(0,p-1)$, and reaches its minimum $a(a-1)$ at $(x_1,x_2)=(0,1)$. Thus, we have $f_a(x_1,x_2) \in [a(a-1), \{a-(p-1)\}a]$.

To obtain the results in Corollary 1, we examine the case when $2-p < q_{\boldsymbol{x}}(1,2-z) < p-1$. From the bounds for $f_{q_{\boldsymbol{x}}(1,2-z)}(p_{\boldsymbol{x}}(1), 1-p_{\boldsymbol{x}}(0))$, we have $r_{\boldsymbol{x}}(z) \in [1, u_{\boldsymbol{x}}(z)]$ where

$$u_{\boldsymbol{x}}(z) = \left\{ \frac{q_{\boldsymbol{x}}(1,2-z)}{q_{\boldsymbol{x}}(1,2-z)-(2-p)}, \frac{1-q_{\boldsymbol{x}}(1,2-z)}{(p-1)-q_{\boldsymbol{x}}(1,2-z)} \right\}$$

According to Theorem 2,

$$\tau(z) = \sum_{\boldsymbol{x}} \Pr(\boldsymbol{X}_i = \boldsymbol{x} \mid D_i = 1) \cdot r_{\boldsymbol{x}}(z) Q_{\boldsymbol{x}}(z),$$

where $Q_{\boldsymbol{x}}(z) = \{m_{\boldsymbol{x}}(1,2-z) - m_{\boldsymbol{x}}(0,2-z)\}\{q_{\boldsymbol{x}}(1,1) - q_{\boldsymbol{x}}(1,2)\}$. Therefore, under $2-p < \min_{\boldsymbol{x}}\{q_{\boldsymbol{x}}(1,2-z)\} \leq \max_{\boldsymbol{x}}\{q_{\boldsymbol{x}}(1,2-z)\} < p-1$, the upper bound of $\tau(z)$ is

$$\sum_{\boldsymbol{x}} \Pr(\boldsymbol{X}_i = \boldsymbol{x} \mid D_i = 1) \cdot \left[I\{Q_{\boldsymbol{x}}(z) \geq 0\} Q_{\boldsymbol{x}}(z) u_{\boldsymbol{x}}(z) + I\{Q_{\boldsymbol{x}}(z) < 0\} Q_{\boldsymbol{x}}(z) \right], \tag{S4}$$

and the upper bound of $\tau(z)$ is

$$\sum_{\boldsymbol{x}} \Pr(\boldsymbol{X}_i = \boldsymbol{x} \mid D_i = 1) \cdot \left[I\{Q_{\boldsymbol{x}}(z) \geq 0\} Q_{\boldsymbol{x}}(z) + I\{Q_{\boldsymbol{x}}(z) < 0\} Q_{\boldsymbol{x}}(z) \right] u_{\boldsymbol{x}}(z). \tag{S5}$$

When Assumption S1 holds instead of Assumption 1, we can replace $\Pr(\boldsymbol{X}_i = \boldsymbol{x} \mid D_i = 1)$ with (S2) in (S4) and (S5) to obtain the bounds. For other cases with different relative magnitude among $p-1$, $2-p$ and $q_{\boldsymbol{x}}(1, 2-z)$, we can obtain bounds for $\tau(z)$ using a similar technique.

S2. Computation

In this section, we provide the details of the EM algorithms for the the proposed sensitivity analyses. We will give the algorithms under Assumption 1. Note that when sensitivity parameter is zero, we obtain the point estimates under Assumptions 1–4. Recall that the following model is fit to the units with $D_i = 1$ ($G_i = c$).

$$\begin{aligned}
Y_{i1}^*(z) &= I(\widetilde{Y}_{i1}(z) > 0) \quad \text{where} \quad \widetilde{Y}_{i1}(z) = g(z, \boldsymbol{X}_i) + \epsilon_{i1}, \\
Y_{i2}(z, y_1^*) &= I(\widetilde{Y}_{i2}(z, y_1^*) > 0) \quad \text{where} \quad \widetilde{Y}_{i2}(z, y_1^*) = f(z, y_1^*, \boldsymbol{X}_i) + \epsilon_{i2}, \\
\begin{pmatrix} \epsilon_{i1} \\ \epsilon_{i2} \end{pmatrix} &\sim \boldsymbol{N}_2 \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \boldsymbol{\Sigma} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right\},
\end{aligned}$$

where $g(\cdot)$ and $f(\cdot)$ have linear forms

$$\begin{aligned}
g(z, \boldsymbol{x}) &= \alpha_0 + \alpha_Z z + \boldsymbol{x}\boldsymbol{\alpha}_X + z\boldsymbol{x}\boldsymbol{\alpha}_{ZX}, \\
f(z, y_1, \boldsymbol{x}) &= \beta_0 + \beta_Z z + \beta_Y y_1 + \beta_{ZY} z y_1 + \boldsymbol{x}\boldsymbol{\beta}_X + z\boldsymbol{x}\boldsymbol{\beta}_{ZX} + y_1\boldsymbol{x}\boldsymbol{\beta}_{YX}.
\end{aligned}$$

Define $\boldsymbol{W}_{i1} = (1, Z_i, \boldsymbol{X}_i, Z_i\boldsymbol{X}_i)^\top$, $\boldsymbol{W}_{i2} = (1, Z_i, Y_{i1}^*, Z_iY_{i1}^*, \boldsymbol{X}_i, Z_i\boldsymbol{X}_i, Y_{i1}^*\boldsymbol{X}_i)^\top$, $\boldsymbol{\alpha} = (\alpha_0, \alpha_Z, \boldsymbol{\alpha}_X, \boldsymbol{\alpha}_{ZX})^\top$, and $\boldsymbol{\beta} = (\beta_0, \beta_Z, \beta_Y, \beta_{ZY}, \boldsymbol{\beta}_X, \boldsymbol{\beta}_{ZX}, \boldsymbol{\beta}_{YX})^\top$. Because $\widetilde{Y}_{i1}(z) = \widetilde{Y}_{i1}$, $Y_{i1}^*(z) = Y_{i1}^*$ and $Y_{i1}(z) = Y_{i1}$ if $Z_i = z$. Similarly, $\widetilde{Y}_{i2}(z, y_1^*) = \widetilde{Y}_{i2}$ and $Y_{i2}(z, y_1) = Y_{i2}$ if $Z_i = z$ and $Y_{i1}^* = y_1^*$. Thus, we can rewrite our model using the observed data:

$$
\begin{aligned}
Y_{i1}^* &= I(\widetilde{Y}_{i1} > 0) \quad \text{where} \quad \widetilde{Y}_{i1} = \boldsymbol{W}_{i1}^\top \boldsymbol{\alpha} + \epsilon_{i1}, \\
Y_{i2} &= I(\widetilde{Y}_{i2} > 0) \quad \text{where} \quad \widetilde{Y}_{i2} = \boldsymbol{W}_{i2}^\top \boldsymbol{\beta} + \epsilon_{i2}, \\
\begin{pmatrix} \epsilon_{i1} \\ \epsilon_{i2} \end{pmatrix} &\sim \boldsymbol{N}_2 \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \boldsymbol{\Sigma} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right\}.
\end{aligned}
$$

S2.1. Sensitivity Analysis for Unobserved Confounding

We present the EM algorithm for the sensitivity analysis regarding unobserved confounding. We write our model as,

$$
\begin{aligned}
Y_{i1}^* &= I(\widetilde{Y}_{i1} > 0) \quad \text{where} \quad \widetilde{Y}_{i1} = \boldsymbol{W}_{i1} \boldsymbol{\alpha} + \epsilon_{i1}, \\
Y_{i2} &= I(\widetilde{Y}_{i2} > 0) \quad \text{where} \quad \widetilde{Y}_{i2} = \boldsymbol{W}_{i2} \boldsymbol{\beta} + \epsilon_{i2}, \\
\begin{pmatrix} \epsilon_{i1} \\ \epsilon_{i2} \end{pmatrix} &\sim \boldsymbol{N}_2 \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \boldsymbol{\Sigma} = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \right\}.
\end{aligned} \tag{S6}
$$

The complete-data log-likelihood function is given by,

$$
\begin{aligned}
\log L_c(\boldsymbol{\alpha}, \boldsymbol{\beta}) = & \sum_{i=1}^N I\{D_i = 1\} \cdot \left[-\frac{(\widetilde{Y}_{i1} - \boldsymbol{W}_{i1}^\top \boldsymbol{\alpha})^2}{2(1-\rho^2)} + \frac{\rho(\widetilde{Y}_{i1} - \boldsymbol{W}_{i1}^\top \boldsymbol{\alpha})(\widetilde{Y}_{i2} - \boldsymbol{W}_{i2}^\top \boldsymbol{\beta})}{1-\rho^2} - \frac{(\widetilde{Y}_{i2} - \boldsymbol{W}_{i2}^\top \boldsymbol{\beta})^2}{2(1-\rho^2)} \right] \\
& \cdot I\{\widetilde{Y}_{i1}(Y_{i1} - 0.5) > 0\} \cdot I\{\widetilde{Y}_{i2}(Y_{i2} - 0.5) > 0\} + constant.
\end{aligned}
$$

Let $\boldsymbol{O}_i$ be the observed data for unit i, i.e., $\boldsymbol{O}_i = (Y_{i1}, Y_{i2}, Z_i, D_i = 1, \boldsymbol{X}_i)^\top$, and let $\boldsymbol{\xi}^{(k)}$ be the estimate of $\boldsymbol{\xi}$ after the k-th iteration. In the E-step, we need to compute:

$$
E(\widetilde{Y}_{i1} \mid \boldsymbol{O}_i, \boldsymbol{\alpha}^{(k)}, \boldsymbol{\beta}^{(k)}), \quad E(\widetilde{Y}_{i2} \mid \boldsymbol{O}_i, \boldsymbol{\alpha}^{(k)}, \boldsymbol{\beta}^{(k)}).
$$

Because $(\widetilde{Y}_{i1}, \widetilde{Y}_{i2})^\top \mid \boldsymbol{O}_i, \boldsymbol{\alpha}^{(k)}, \boldsymbol{\beta}^{(k)}$ follows a truncated bivariate Normal distribution with mean $(\boldsymbol{W}_{i1}^\top \boldsymbol{\alpha}, \boldsymbol{W}_{i2}^\top \boldsymbol{\beta})$ and covariance matrix $\boldsymbol{\Sigma}_2$, we use R package tmvtnorm to compute them.

In the M-step, we need to update the parameters based on

$$\widetilde{Y}_{i1} \;=\; \boldsymbol{W}_{i1}^{\top}\boldsymbol{\alpha} + \epsilon_{i1}, \quad \widetilde{Y}_{i2} = \boldsymbol{W}_{i2}^{\top}\boldsymbol{\beta} + \epsilon_{i2}.$$

Because we know the covariance matrix of the error terms $(\epsilon_{i1}, \epsilon_{i2})$, we can transform the two regression equations to

$$\boldsymbol{\Sigma}_2^{-1/2}\begin{pmatrix}\widetilde{Y}_{i1}\\ \widetilde{Y}_{i2}\end{pmatrix} = \boldsymbol{\Sigma}_2^{-1/2}\begin{pmatrix}\boldsymbol{W}_{i1} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{W}_{i2}\end{pmatrix}\begin{pmatrix}\boldsymbol{\alpha}\\ \boldsymbol{\beta}\end{pmatrix} + \boldsymbol{\Sigma}_2^{-1/2}\begin{pmatrix}\epsilon_{i1}\\ \epsilon_{i2}\end{pmatrix}.$$

Then, we can use ordinary least squares regression to update the parameters.

After obtaining the maximum likelihood estimates of the parameters, we can then write,

$$\begin{aligned}
&P\{Y_{i2}(z, Y_{i1}^*(z')) = 1 \mid G_i = c\} = \sum_{\boldsymbol{x}} P\{Y_{i2}(z, Y_{i1}^*(z')) = 1 \mid \boldsymbol{X}_i = \boldsymbol{x}, D_i = 1\}\mathrm{pr}(\boldsymbol{X}_i = \boldsymbol{x} \mid D_i = 1)\\
= &\sum_{\boldsymbol{x}}[P\{Y_{i2}(z,1) = 1, Y_{i1}^*(z') = 1 \mid \boldsymbol{X}_i = \boldsymbol{x}, D_i = 1\} + P\{Y_{i2}(z,0) = 1, Y_{i1}^*(z') = 0 \mid \boldsymbol{X}_i = \boldsymbol{x}, D_i = 1\}]\\
&\quad\cdot\mathrm{pr}(\boldsymbol{X}_i = \boldsymbol{x} \mid D_i = 1)\\
= &\sum_{\boldsymbol{x}}[P\{\epsilon_{i2} > -f(z,1,\boldsymbol{x}), \epsilon_{i1} > -g(z,\boldsymbol{x}) \mid \boldsymbol{X}_i = \boldsymbol{x}, D_i = 1\}\\
&\quad + P\{\epsilon_{i2} > -f(z,0,\boldsymbol{x}), \epsilon_{i1} \le -g(z,\boldsymbol{x}) \mid \boldsymbol{X}_i = \boldsymbol{x}, D_i = 1\}]\mathrm{pr}(\boldsymbol{X}_i = \boldsymbol{x} \mid D_i = 1).
\end{aligned}$$

We calculate the terms above using the cumulative distribution function of bivariate Normal distributions. Then, based on $P\{Y_{i2}(z, Y_{i1}^*(z')) = 1 \mid G_i = c\}$, we compute the estimated average contagion and direct effects.

S2.2. Sensitivity Analysis for Additive Measurement Error

We assume

$$Y_{i1}(z) \;=\; I\{\widetilde{Y}_{i1}(z) + \zeta_i > 0\} \quad \text{and} \quad \begin{pmatrix}\zeta_i\\ \epsilon_{i2}\end{pmatrix} \overset{i.i.d.}{\sim} \boldsymbol{N}_2\left\{\begin{pmatrix}0\\ 0\end{pmatrix}, \boldsymbol{\Sigma} = \begin{pmatrix}\sigma^2 & \rho_e\sigma\\ \rho_e\sigma & 1\end{pmatrix}\right\}.$$

where σ^2 and ρ_e are pre-specified.

Therefore, we can write the model as:

$$
\begin{aligned}
&Y_{i1}^* = I(Y_{i1}' - \zeta_i > 0), \quad Y_{i1} = I(Y_{i1}' > 0) \quad \text{where} \quad Y_{i1}' = \boldsymbol{W}_{i1}^\top \boldsymbol{\alpha} + \epsilon_{i1}', \\
&Y_{i2} = I(\widetilde{Y}_{i2} > 0) \quad \text{where} \quad \widetilde{Y}_{i2} = \boldsymbol{W}_{i2}^\top \boldsymbol{\beta} + \epsilon_{i2}, \\
&\zeta_i \sim N(0, \sigma^2), \quad \begin{pmatrix} \epsilon_{i1}' \\ \epsilon_{i2} \end{pmatrix} \sim \boldsymbol{N}_2 \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \boldsymbol{\Sigma}_1 = \begin{pmatrix} 1+\sigma^2 & \rho_e \sigma \\ \rho_e \sigma & 1 \end{pmatrix} \right\}.
\end{aligned} \tag{S7}
$$

Treating Y_{i1}', Y_{i1}^* and $\widetilde{Y}_{i2}$ as missing data, we can write the complete-data log-likelihood for the units with $D_i = 1$ as

$$
\begin{aligned}
&\log L_c(\boldsymbol{\xi}) \\
= &\sum_{i=1}^N I(D_i = 1) \cdot \left[-\frac{1}{1-\rho_e'^2} \left\{ \frac{(Y_{i1}' - \boldsymbol{W}_{i1}^\top \boldsymbol{\alpha})^2}{2(1+\sigma^2)} - \frac{\rho_e'(Y_{i1}' - \boldsymbol{W}_{i1}^\top \boldsymbol{\alpha})(\widetilde{Y}_{i2} - \boldsymbol{W}_{i2}^\top \boldsymbol{\beta})}{\sqrt{1+\sigma^2}} + \frac{(\widetilde{Y}_{i2} - \boldsymbol{W}_{i2}^\top \boldsymbol{\beta})^2}{2} \right\} \right. \\
&\left. + h(Y_{i1}^*, Y_{i1}') \right] \cdot I\{Y_{i1}(Y_{i1}' - 0.5) > 0\} I\{\widetilde{Y}_{i2}(Y_{i2} - 0.5) > 0\} + constant,
\end{aligned}
$$

where $\rho_e' = \rho_e \sigma / \sqrt{1+\sigma^2}$, and $h(Y_{i2}^*, Y_{i2}')$ is the likelihood that corresponds to $\text{pr}(Y_{i2}^* \mid Y_{i2}')$ which does not affect our parameter estimation.

We use the EM algorithm to obtain the MLEs of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$. We ignore $D_i = 1$ in the following derivation. In the M-step, we update the parameters conditionally. In particular, we update $\boldsymbol{\alpha}$ conditional on $\boldsymbol{\beta}$:

$$
\begin{aligned}
\boldsymbol{\alpha}^{(k+1)} &= \left\{ \sum_{i=1}^N E(\boldsymbol{W}_{i1} \boldsymbol{W}_{i1}^\top \mid \boldsymbol{O}_i, \boldsymbol{\xi}^{(k)}) \right\}^{-1} \left\{ \sum_{i=1}^N E\left(\boldsymbol{W}_{i1} \widetilde{Y}_{i1}' - \rho_e' \sqrt{1+\sigma^2} \boldsymbol{W}_{i1} (\widetilde{Y}_{i2} - \boldsymbol{W}_{i2}^\top \boldsymbol{\beta}^{(k)}) \mid \boldsymbol{O}_i, \boldsymbol{\xi}^{(k)} \right) \right\} \\
&= \left\{ \sum_{i=1}^N E(\boldsymbol{W}_{i1} \boldsymbol{W}_{i1}^\top \mid \boldsymbol{O}_i, \boldsymbol{\xi}^{(k)}) \right\}^{-1} \left\{ \sum_{i=1}^N E\left(\boldsymbol{W}_{i1} \widetilde{Y}_{i1}' - \rho_e \sigma \boldsymbol{W}_{i1} (\widetilde{Y}_{i2} - \boldsymbol{W}_{i2}^\top \boldsymbol{\beta}^{(k)}) \mid \boldsymbol{O}_i, \boldsymbol{\xi}^{(k)} \right) \right\}
\end{aligned} \tag{S8}
$$

and update $\boldsymbol{\beta}$ conditional on $\boldsymbol{\alpha}$:

$$
\begin{aligned}
\boldsymbol{\beta}^{(k+1)} &= \left\{ \sum_{i=1}^N E(\boldsymbol{W}_{i2} \boldsymbol{W}_{i2}^\top \mid \boldsymbol{O}_i, \boldsymbol{\xi}^{(k)}) \right\}^{-1} \left\{ \sum_{i=1}^N E\left(\boldsymbol{W}_{i2} \widetilde{Y}_{i2} - \frac{\rho_e' \boldsymbol{W}_{i2} (Y_{i1}' - \boldsymbol{W}_{i1}^\top \boldsymbol{\alpha}^{(k+1)})}{\sqrt{1+\sigma^2}} \mid \boldsymbol{O}_i, \boldsymbol{\xi}^{(k)} \right) \right\} \\
&= \left\{ \sum_{i=1}^N E(\boldsymbol{W}_{i2} \boldsymbol{W}_{i2}^\top \mid \boldsymbol{O}_i, \boldsymbol{\xi}^{(k)}) \right\}^{-1} \left\{ \sum_{i=1}^N E\left(\boldsymbol{W}_{i2} \widetilde{Y}_{i2} - \frac{\rho_e \sigma \boldsymbol{W}_{i2} (Y_{i1}' - \boldsymbol{W}_{i1}^\top \boldsymbol{\alpha}^{(k+1)})}{1+\sigma^2} \mid \boldsymbol{O}_i, \boldsymbol{\xi}^{(k)} \right) \right\}.
\end{aligned} \tag{S9}
$$

Therefore, we need to calculate the following conditional expectations in the E-step:

$$
\begin{aligned}
&E(Y_{i1}^{*2} \mid \boldsymbol{O}_i, \boldsymbol{\xi}^{(k)}) = E(Y_{i1}^{*} \mid \boldsymbol{O}_i, \boldsymbol{\xi}^{(k)}), \quad E(Y_{i1}' Y_{i1}^{*} \mid \boldsymbol{O}_i, \boldsymbol{\xi}^{(k)}) \\
&E(Y_{i1}' \mid \boldsymbol{O}_i, \boldsymbol{\xi}^{(k)}), \quad E(\widetilde{Y}_{i2} Y_{i1}^{*} \mid \boldsymbol{O}_i, \boldsymbol{\xi}^{(k)}), \quad E(\widetilde{Y}_{i2} \mid \boldsymbol{O}_i, \boldsymbol{\xi}^{(k)}).
\end{aligned}
$$

We calculate them separately. First, given Y_{i1}^*, Y_{i1} and Y_{i2}, $(Y_{i1}' - \zeta_i, Y_{i1}', \widetilde{Y}_{i2})$ follows a trivariate truncated Normal distribution:

$$
\begin{pmatrix} Y_{i1}' - \zeta_i \\ Y_{i1}' \\ \widetilde{Y}_{i2} \end{pmatrix} \sim \boldsymbol{TN}_3 \left\{ \begin{pmatrix} \boldsymbol{W}_{i1}\boldsymbol{\alpha} \\ \boldsymbol{W}_{i1}\boldsymbol{\alpha} \\ \boldsymbol{W}_{i2}\boldsymbol{\beta} \end{pmatrix}, \boldsymbol{\Sigma}_1 = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1+\sigma^2 & \rho_e\sigma \\ 0 & \rho_e\sigma & 1 \end{pmatrix} \right\},
$$

where the truncated intervals are given by $Y_{i1}^* = I(Y_{i1}' - \zeta_i > 0)$, $Y_{i1} = I(Y_{i1}' > 0)$ and $Y_{i2} = I(\widetilde{Y}_{i2} > 0)$. By Bayes Theorem, we have

$$
\begin{aligned}
&E(Y_{i1}^{*2} \mid \boldsymbol{O}_i, \boldsymbol{\xi}^{(k)}) = \mathrm{pr}(Y_{i1}^* = 1 \mid Z_i, D_i = 1, \boldsymbol{X}_i, \boldsymbol{\xi}^{(k)}) \\
= \; & \frac{\mathrm{pr}(Y_{i1}^* = 1, Y_{i1}, Y_{i2} \mid \boldsymbol{O}_i, \boldsymbol{\xi}^{(k)})}{\mathrm{pr}(Y_{i1}^* = 1, Y_{i1}, Y_{i2} \mid Z_i, D_i = 1, \boldsymbol{X}_i, \boldsymbol{\xi}^{(k)}) + \mathrm{pr}(Y_{i1}^* = 0, Y_{i1}, Y_{i2} \mid Z_i, D_i = 1, \boldsymbol{X}_i, \boldsymbol{\xi}^{(k)})},
\end{aligned}
$$

where $\mathrm{pr}(Y_{i1}^*, Y_{i1}, Y_{i2} \mid Z_i, D_i = 1, \boldsymbol{X}_i, \boldsymbol{\xi}^{(k)})$ can be calculated from R package tmvtnorm, which contains functions for calculating cumulative distribution functions and expectations for multivariate truncated Normal distributions. Using this package, we can then calculate

$$
\begin{aligned}
E(Y_{i1}' Y_{i1}^* \mid \boldsymbol{O}_i, \boldsymbol{\xi}^{(k)}) &= E(Y_{i1}' \mid Y_{i1}^* = 1, \boldsymbol{O}_i, \boldsymbol{\xi}^{(k)}) \cdot \mathrm{pr}(Y_{i1}^* = 1 \mid \boldsymbol{O}_i, \boldsymbol{\xi}^{(k)}), \\
E(Y_{i1}' \mid \boldsymbol{O}_i, \boldsymbol{\xi}^{(k)}) &= E(Y_{i1}' Y_{i1}^* \mid \boldsymbol{O}_i, \boldsymbol{\xi}^{(k)}) + E(Y_{i1}'(1 - Y_{i1}^*) \mid \boldsymbol{O}_i, \boldsymbol{\xi}^{(k)}), \\
E(\widetilde{Y}_{i2} Y_{i1}^* \mid \boldsymbol{O}_i, \boldsymbol{\xi}^{(k)}) &= E(\widetilde{Y}_{i2} \mid Y_{i1}^* = 1, \boldsymbol{O}_i, \boldsymbol{\xi}^{(k)}) \cdot \mathrm{pr}(Y_{i1}^* = 1 \mid \boldsymbol{O}_i, \boldsymbol{\xi}^{(k)}) \\
E(\widetilde{Y}_{i2} \mid \boldsymbol{O}_i, \boldsymbol{\xi}^{(k)}) &= E(\widetilde{Y}_{i2} Y_{i1}^* \mid \boldsymbol{O}_i, \boldsymbol{\xi}^{(k)}) + E(\widetilde{Y}_{i2}(1 - Y_{i1}^*) \mid \boldsymbol{O}_i, \boldsymbol{\xi}^{(k)}).
\end{aligned}
$$

Based on these conditional expectations, we can update the parameters using (S8) and (S9).

After obtaining the maximum likelihood estimates of the parameters, we then write,

$$
\begin{aligned}
& P\{Y_{i2}(z, Y^*_{i1}(z')) = 1 \mid G_i = c\} = \sum_{\boldsymbol{x}} P\{Y_{i2}(z, Y^*_{i1}(z')) = 1 \mid \boldsymbol{X}_i = \boldsymbol{x}, D_i = 1\}\mathrm{pr}(\boldsymbol{X}_i = \boldsymbol{x} \mid D_i = 1) \\
= & \sum_{\boldsymbol{x}} [P\{Y_{i2}(z, 1) = 1, Y^*_{i1}(z') = 1 \mid \boldsymbol{X}_i = \boldsymbol{x}, D_i = 1\} + P\{Y_{i2}(z, 0) = 1, Y^*_{i1}(z') = 0 \mid \boldsymbol{X}_i = \boldsymbol{x}, D_i = 1\}] \\
& \quad \cdot \mathrm{pr}(\boldsymbol{X}_i = \boldsymbol{x} \mid D_i = 1) \\
= & \sum_{\boldsymbol{x}} [P\{\epsilon_{i2} > -f(z, 1, \boldsymbol{x}), \epsilon_{i1} > -g(z, \boldsymbol{x}) \mid \boldsymbol{X}_i = \boldsymbol{x}, D_i = 1\} \\
& \quad + P\{\epsilon_{i2} > -f(z, 0, \boldsymbol{x}), \epsilon_{i1} \leq -g(z, \boldsymbol{x}) \mid \boldsymbol{X}_i = \boldsymbol{x}, D_i = 1\}]\mathrm{pr}(\boldsymbol{X}_i = \boldsymbol{x} \mid D_i = 1).
\end{aligned}
$$

We calculate the terms above using the cumulative distribution function of bivariate Normal distributions. Then, based on $P\{Y_{i2}(z, Y^*_{i1}(z')) = 1 \mid G_i = c\}$, we compute the estimated average contagion and direct effects.