Using Algorithms to Detect Gerrymandering: ALgorithm-Assisted Redistricting Methodology (ALARM) Project

Kosuke Imai

Harvard University

Department of Political Science, Washington University, St. Louis

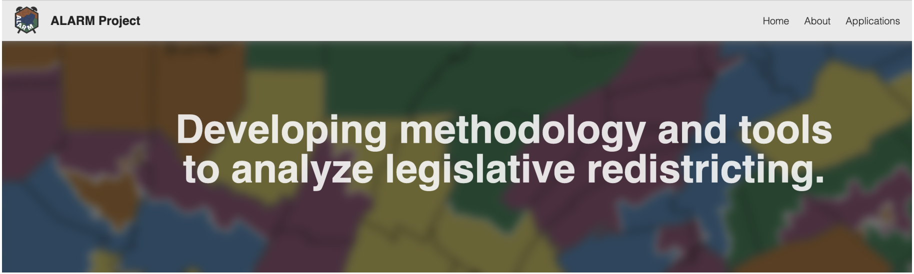
February 9, 2023

Joint work with Christopher Kenny, Cory McCartan, Tyler Simko, Shiro Kuriwaki, George Garcia III, Kevin Wang, and Melissa Wu

Motivation

- Today's world for quantitative social science:
  1. increasing availability of granular data
  2. rapid methodological advancement
- Social scientists can and should solve problems of the real world!
- Redistricting as a major policy decision
- How can we use data and algorithms to evaluate redistricting plans?
  - traditional methods: comparison across states and time periods
  - confounded by state-specific political geography and rules
- Use of simulation algorithms
  1. obtain a representative sample of redistricting plans under constraints
  2. compare the enacted plan with this baseline distribution
- Technological solution to detecting gerrymandering
- Tool for analyzing redistricting

ALgorithm-Assisted Redistricting Methodology (ALARM)

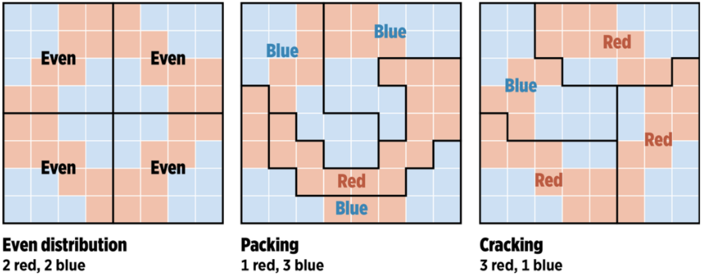


- What we do:
  1. develop efficient and flexible simulation algorithms
  2. build open-source software packages for the entire workflow
  3. evaluate redistricting plans in the United States and elsewhere

- Goal: empower researchers, policy makers, data journalists, and citizen data scientists with powerful tools

Redistricting Basics

- Classic gerrymandering strategies: packing and cracking



- What has changed:
  - availability of granular data
  - mapping software (e.g., Maptitude, Dave's Redistricting app)
- US Congressional redistricting
  - racial gerrymandering: *Shelby County v. Holder*; *Merrill v. Milligan*
  - partisan gerrymandering: *Rucho v. Common Cause*; *Moore v. Harper*

Why Use Simulation Algorithm for Redistricting Evaluation?

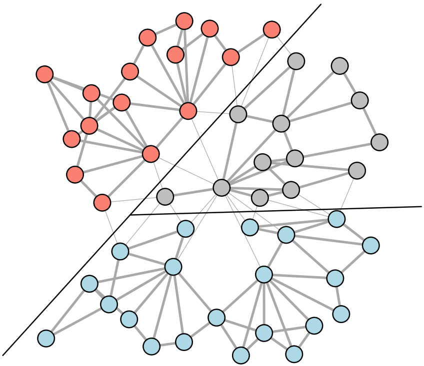
- Traditional redistricting evaluation
  1. compute various fairness metrics
  2. compare them across states and over time
- Confounded by differences in political geography and redistricting rules

- Simulation-based redistricting evaluation
  1. generate many alternative plans under a set of redistricting criteria
  2. compare them with a proposed plan to evaluate its properties

- Benefits of simulation approach
  1. can control for state-specific political geography and redistricting rules
  2. transparency and ability to isolate a relevant factor
  3. mathematical properties $\rightsquigarrow$ representative sample of alternative plans

Redistricting as a Balanced Graph Partition Problem

50 Precincts, Three Districts



- Efficient enumeration algorithm exists (Fifield *et al.* 2020)
- Only applicable to very small redistricting problems
- $9 \times 9 \rightarrow 9$: equal size $\approx$ 700 trillion, unqual size $\approx$ 160 septillion

Existing Algorithms

1. Constructive Monte Carlo (Chen & Rodden, 2013; Magleby & Mosesson, 2018)
    - randomly select "seeds" and grow districts
    - unknown target population

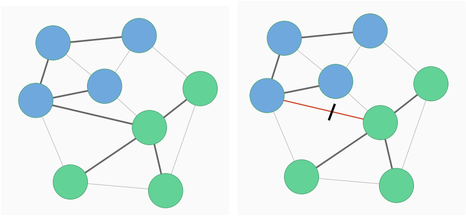
2. Flip algorithms (Fifield *et al.*, 2020; Mattingly & Vaughn, 2014; Chikina *et al.* 2017)
    - start with a valid plan and then reassign units on district boundaries
    - target distribution

$$\pi(\xi) \propto \underbrace{\exp(-J(\xi))}_{\text{custom constraints}} \times \underbrace{1_{\xi \text{ connected}}}_{\text{contiguity requirement}} \times \underbrace{1_{\text{dev}(\xi) \leq D}}_{\text{population balance}}$$

    - incremental changes; applicable for local exploration
    - does not scale; compactness needs to be specified in $J(\cdot)$

2. Merge-split algorithms (DeFord *et al.*, 2021; Carter *et al.* 2019)

- randomly choose a pair of adjacent districts, merge them, and split them into two new districts using uniform spanning trees



- target distribution

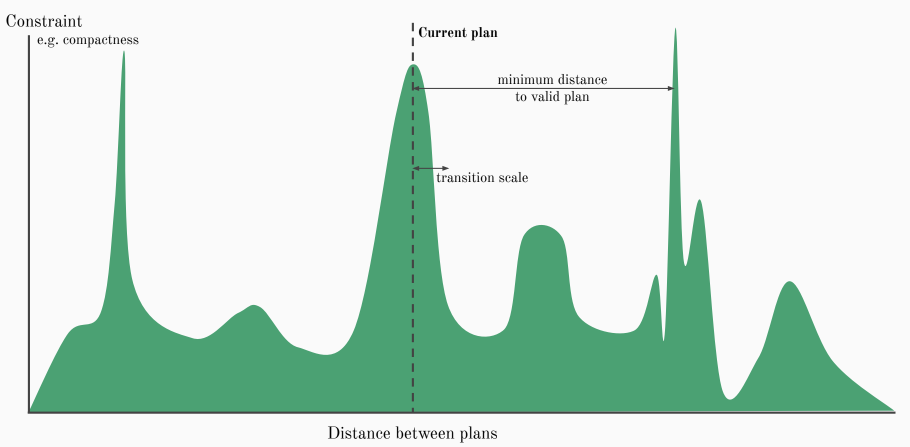
$$\pi(\xi) \propto \underbrace{\tau(\xi)^{\rho}}_{\text{compactness}} \exp(-J(\xi)) \times 1_{\xi \text{ connected}} \times 1_{\text{dev}(\xi) \leq D}$$

  where $\tau(\xi)$ counts the product of the number of spanning trees in each district of the plan ξ

- relation with edge removal compactness

$$\tau(\xi)^{\rho} \approx C_1 \exp(-C_2 \, \rho \, \text{rem}(\xi)) \quad \text{where} \quad \text{rem}(\xi) = \underbrace{1 - \frac{\sum_{i=1}^{n} |E_i(\xi)|}{|E(G)|}}_{\text{fraction of edges removed}}$$

Challenge of MCMC Algorithms



- simulated annealing, parallel tempering $\rightsquigarrow$ difficult to apply in practice

Sequential Monte Carlo (SMC) Algorithm (McCartan and Imai, 2020)

- Start with a blank state in parallel, use the spanning tree approach to sample a district at a time, resample with weights at each step

- Advantage: unlike MCMC, sampled plans are nearly independent
- Limitation: hard to incorporate plan-wide or region-specific constraints

The Splitting Procedure

1. Generate a uniform spanning tree (Wilson's algorithm)
2. Sort edges by population deviation
3. Sample one edge from top k_i edges and remove it
4. Check population bounds

- Probability of splitting a new district G_i from $\widetilde{G}_{i-1}$:

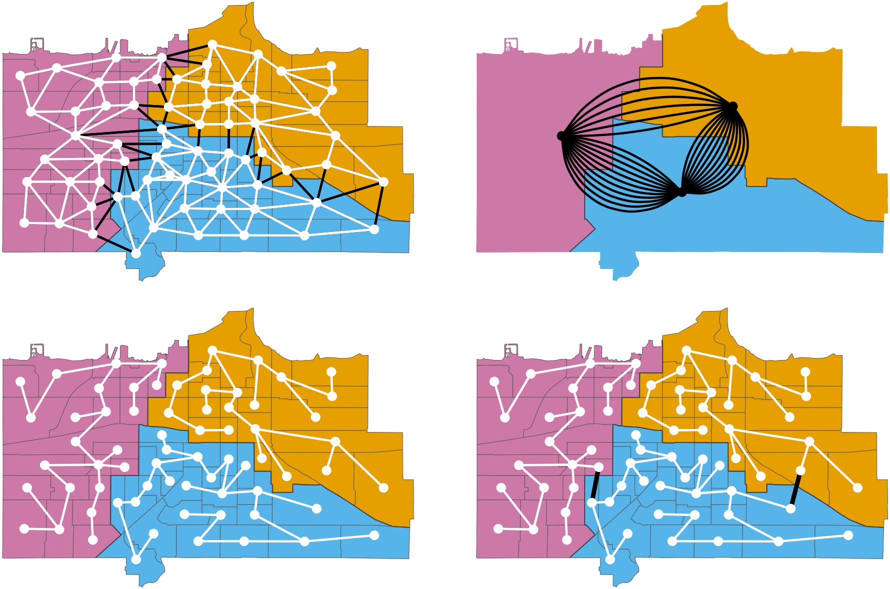
$$\frac{\overbrace{\tau(G_i)\tau(\widetilde{G}_i)}^{\text{all possible ways to connect}}}{\underbrace{\tau(\widetilde{G}_{i-1})k_i}_{\text{all possible ways to split}}} \times \underbrace{|\mathcal{C}(G_i, \widetilde{G}_i)|}_{\text{number of connecting edges}}$$

- Use these probabilities to construct weights

The SMC Algorithm

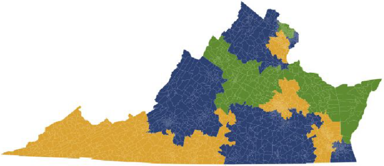
1. Generate S initial copies of map; set all weights to 1
2. For $i \in \{1, 2, \ldots, n-1\}$:
    a. Until there are S successes
        i. Sample a map according to the weights
        ii. Split off a new district from each sampled map
        iii. Reject if population bounds are not met
    b. Calculate new weights based on splitting probability
3. Output complete plans and compute final weights

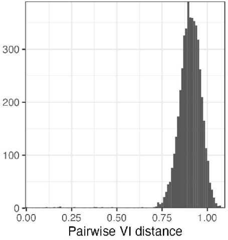
Avoiding County Splits through Quotient Multigraph



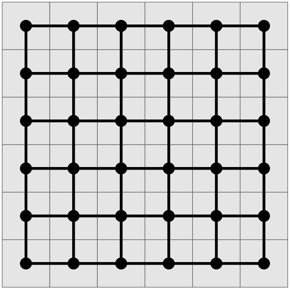
SMC Diagnostics

```
SMC: 1,000 sampled plans of 11 districts on 2,465 units
`adapt_k_thresh`=0.985 • `seq_alpha`=0.5
`est_label_mult`=1 • `pop_temper`=0.01

Plan diversity 80% range: 0.82 to 0.98

R-hat values for summary statistics:
pop_overlap        comp         dem       e_dem
     1.0234      1.0112      1.0053      1.0042

Sampling diagnostics for SMC run 1 of 4 (250 samples)
         Eff. samples (%) Acc. rate Log wgt. sd  Max. unique Est. k
Split 1       242 (97.0%)     20.6%        0.36  245 ( 98%)     10
Split 2       240 (95.8%)     31.2%        0.43  193 ( 77%)      6
Split 3       233 (93.4%)     21.8%        0.49  199 ( 80%)      8
Split 4       231 (92.3%)     29.9%        0.56  196 ( 78%)      5
Split 5       219 (87.6%)     36.1%        0.62  195 ( 78%)      3
Split 6       213 (85.0%)     44.9%        0.67  191 ( 76%)      2
Split 7       224 (89.7%)     15.9%        0.59  189 ( 76%)      7
Split 8       227 (90.8%)     24.2%        0.59  192 ( 77%)      4
Split 9       227 (90.9%)     16.9%        0.60  181 ( 72%)      3
Split 10      228 (91.3%)      3.8%        0.58  174 ( 70%)      2
Resample      166 (66.4%)       NA%        0.59  183 ( 73%)     NA
```

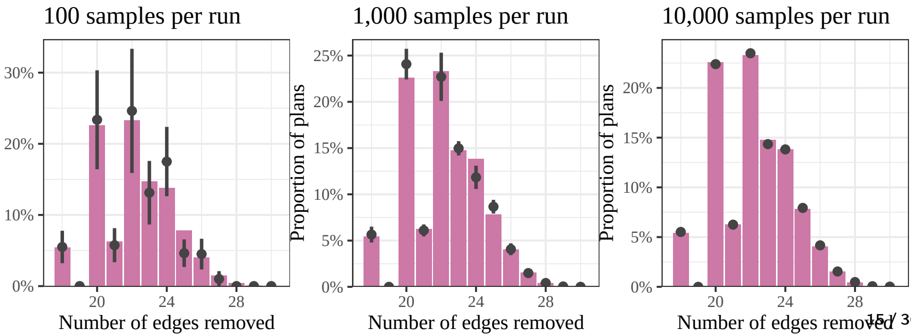




Validation



- Divide a 6×6 grid into 6 equal-sized districts
- Enumerate 451,206 plans (out of 356 billion)
- Number of edge removed as a target statistic



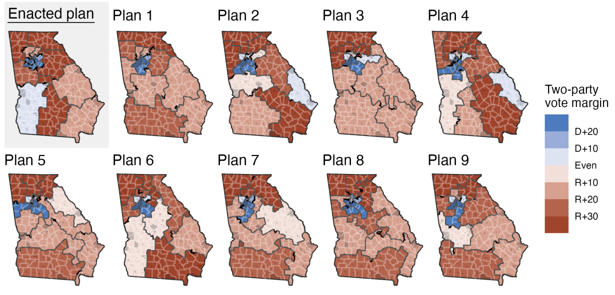
50 State Redistricting Simulations Project

Comprehensive project to simulate alternative congressional redistricting plans for all fifty states.

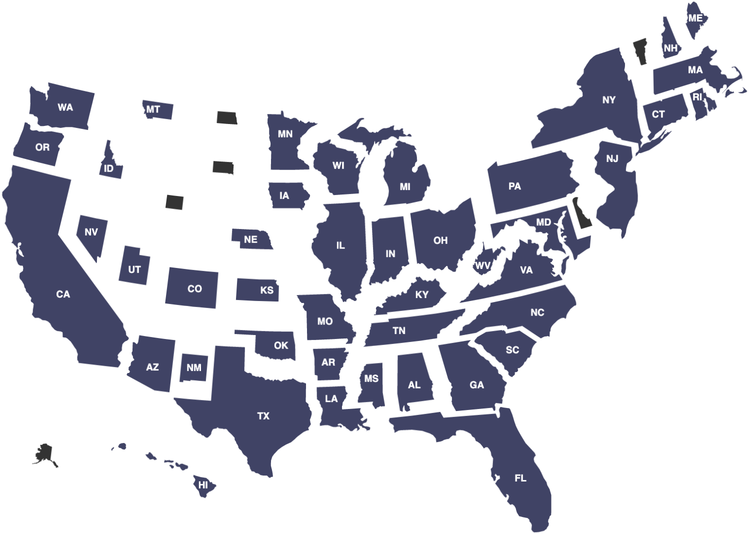
- tidied 2020 Census plus statewide election data from the VEST
- collect state-specific redistricting requirements
- construct algorithmic constraints based on these and traditional redistricting criteria
- 5,000 simulation plans based on SMC
- code and data are available at the Harvard Dataverse

Georgia Example

- 14 Congressional districts
- According to Georgia's House Legislative and Congressional Reapportionment Committee, districts must:
  1. be contiguous
  2. have equal populations
  3. be geographically compact
  4. preserve county and municipality boundaries as much as possible
  5. avoid the unnecessary pairing of incumbents
- We attempted to account for everything except incumbency constraint
- Voting rights act (VRA) compliance is tricky



Check out https://alarm-redist.org/fifty-states/



States colored blue have enacted a congressional map and been fully analyzed, states colored gray have enacted a plan but haven't yet been analyzed, or just have a single district (and hence no redistricting), and states colored red haven't enacted a plan yet.

Electoral Modeling

- Precinct-level data from the 2016 and 2020 presidential elections
- Average of the two elections for precinct $j \rightsquigarrow$ baseline partisanship

$$\hat{D}_j = \underbrace{\frac{1}{2}\left(\frac{D_{16j}}{D_{16j}+R_{16j}} + \frac{D_{20j}}{D_{20j}+R_{20j}}\right)}_{\text{average Democratic vote share}} \times \underbrace{\sqrt{(D_{16j}+R_{16j})(D_{20j}+R_{20j})}}_{\text{(geometric) average turnout}}$$

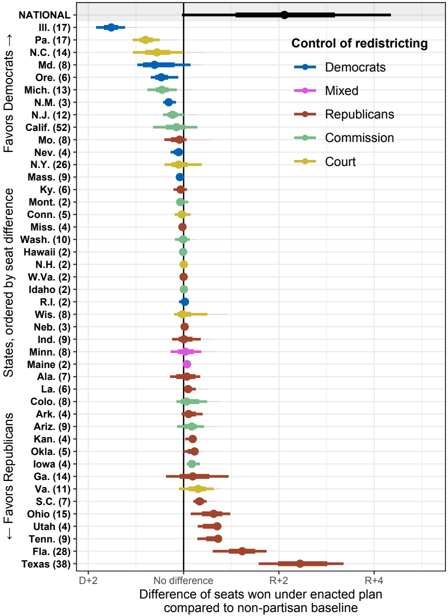
- Model for the Democratic vote share y_{it} for district i in election t:

$$\operatorname{logit}(y_{it}) = \alpha_i + \beta_t + \varepsilon_{it},$$
$$\beta_t \overset{iid}{\sim} \mathcal{N}(0, \sigma_\beta^2), \quad \varepsilon_{it} \overset{iid}{\sim} \mathrm{t}_\nu(0, \sigma_\varepsilon^2),$$

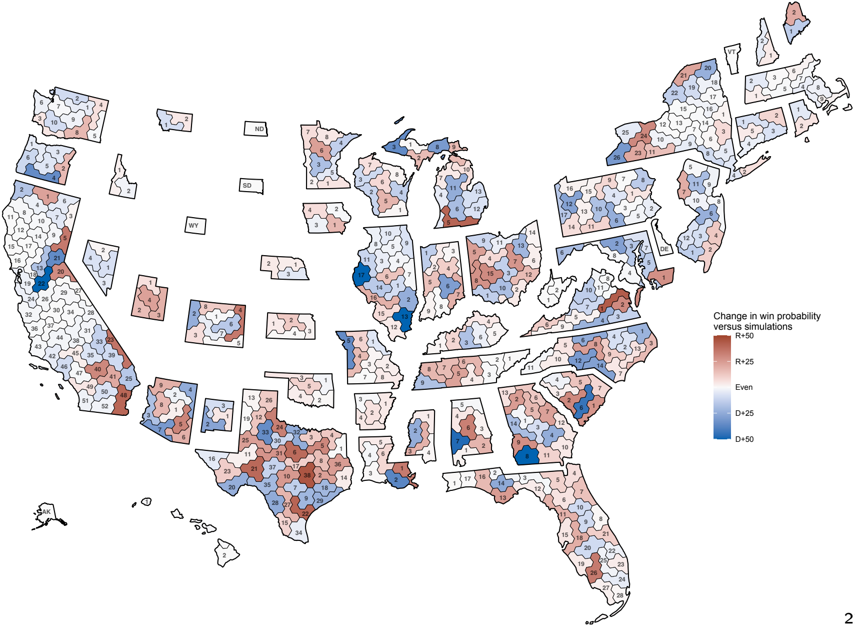
  where we fix α_i to the baseline Democratic vote share and $(\sigma_\beta^2, \sigma_\epsilon^2, \nu)$ are estimated using the historical House elections data since 1976

- We then compute α_i using $\hat{D}_j$ and simulate $(\beta_t, \varepsilon_{it})$ under a given (enacted or simulated) redistricting plan

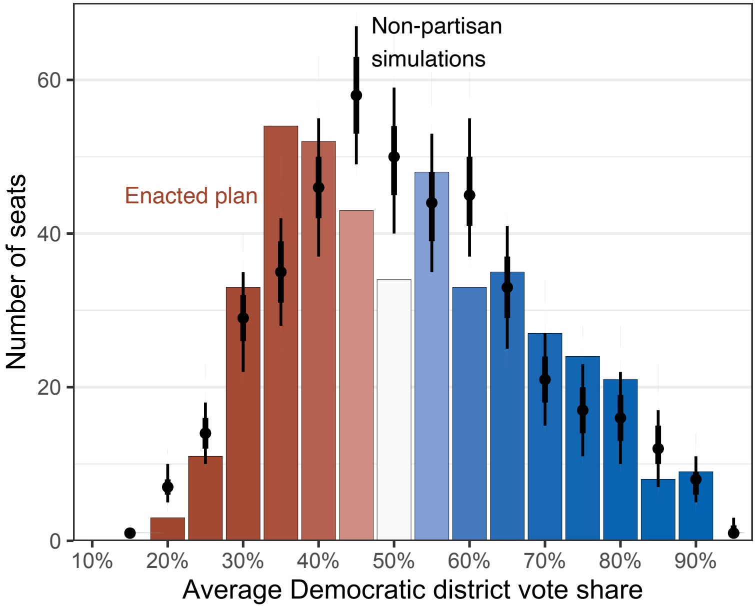
Widespread Partisan Gerrymandering Cancels Nationally



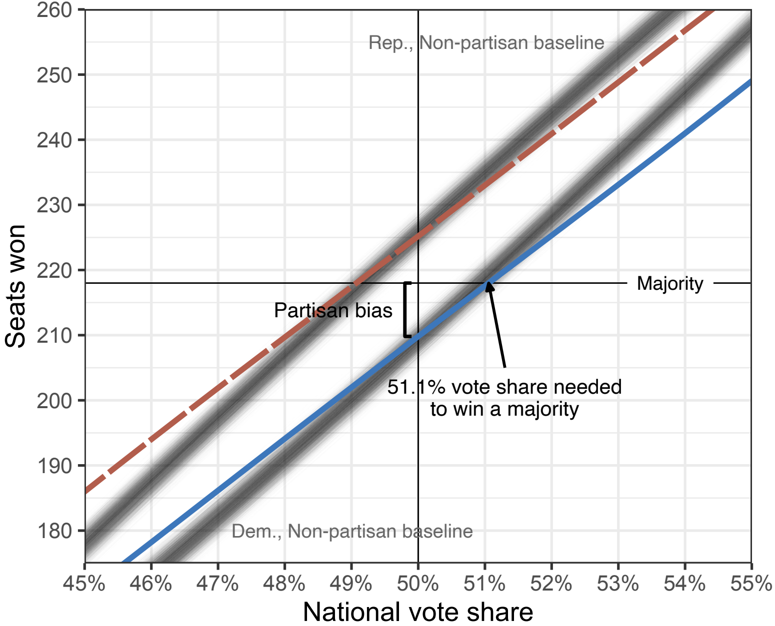
Map of Partisan Gerrymandering



Partisan Gerrymandering Reduces Competitiveness



Partisan Gerrymandering Reduces Responsiveness



Application in the Court: Ohio Congressional Redistricting

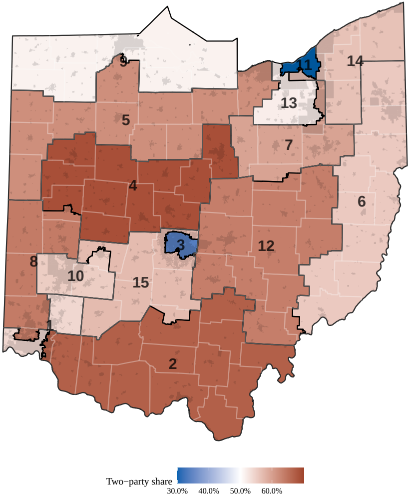
- Currently 16 districts: 4 Democrats and 12 Republicans
  - 2020 President: Biden 45%, Trump 53%
  - 2018 Senate: Brown 53%, Renacci 47%
- After 2020 Census, the number of seats is reduced to 15 districts
- 2018 Ohio voters passed the constitutional amendment
  - bipartisan support leads to a 10 year map
  - if that fails, it becomes a 4 year map
- Redistricting
  - State Senate and House approved the initial map
  - No bipartisan support $\rightsquigarrow$ 4 year map
  - November 20: Governor DeWine signed the map

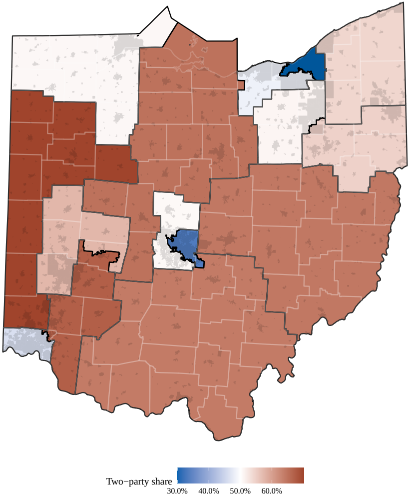
League of Women Voters of Ohio *et al. v.* Ohio Redistricting Commission, *et al.*

- I served as an expert witness for Relators

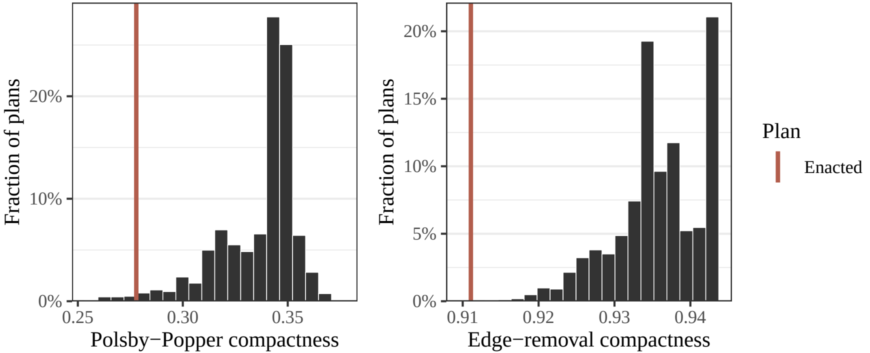
- Simulation analysis based on Sequential Monte Carlo algorithm
  - 5,000 alternative plans
  - contiguous and compact districts
  - compliant with the Voting Rights Act (Cleveland)
  - several complicated splitting constraints
  - Section 2(B)(5): out of Ohio's 88 counties,
    - at least 65 counties should not be split
    - no more than 18 counties can be split no more than once
    - no more than 5 counties can be split no more than twice

The Enacted and Example Simulated Plans



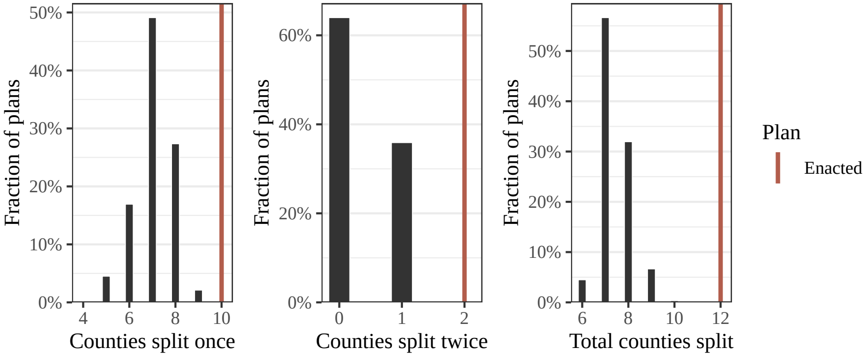


Compactness

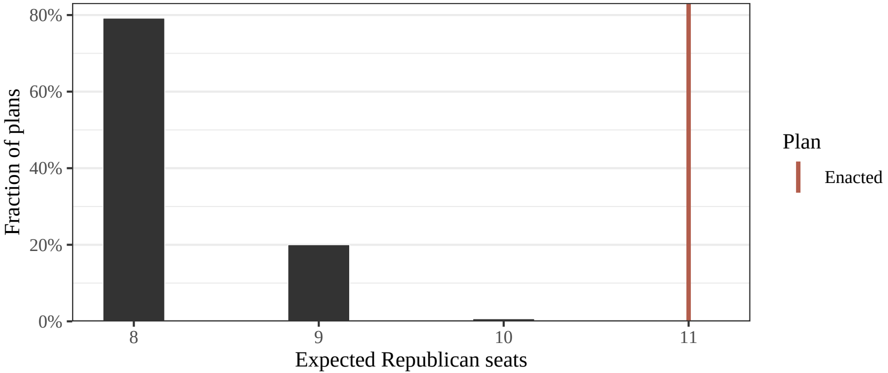


- Polsby-Popper: the ratio of the district area to the area of a circle with the same perimeter
- Edge-removal

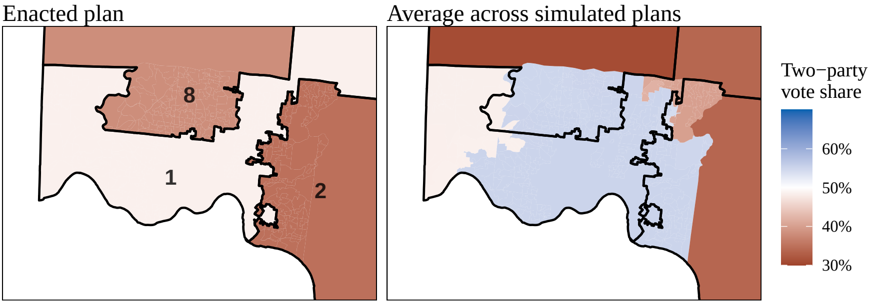
Administrative Boundary Splits



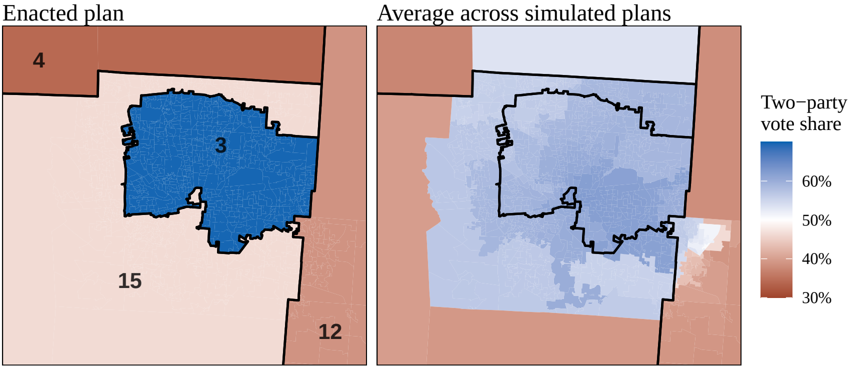
Expected Number of Republican Seats



Cracking: Hamilton County (Cincinnati Area)



Packing: Franklin County (Columbus Area)



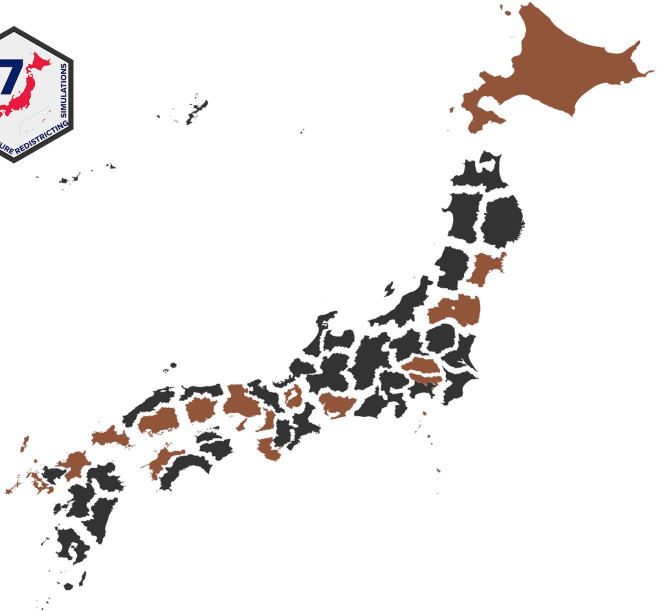
Ohio Supreme Court Strikes Down the Enacted Map

The Court Opinion

Id. at Section 1(C)(3)(a). The above evidence, particularly Dr. Imai's conclusion that the enacted plan will result in, on average, 2.8 more Republican seats than are warranted, shows that the General Assembly's decision to shift what could have been—under a neutral application of Article XIX—Democratic-leaning areas into competitive districts, i.e., districts that give the Republican Party's candidates a better chance of winning than they would otherwise have had in a more compactly drawn district, resulted in a plan that unduly favors the Republican Party and unduly disfavors the Democratic Party.

JUSTICE SOTOMAYOR: I find it interesting that you're touting Dr. Imai's studies when, below, you vehemently objected to his studies on the basis that the studies were incomplete and didn't take into account all of Alabama's guidelines.

Beyond the United States: Japanese Redistricting



Concluding Remarks

- Redistricting matters
  - fair representation and policy outcomes
  - competitiveness of districts and responsiveness
  - political polarization
  - state and local offices, education districts, non-US contexts
- How should we stop gerrymandering?
  - independent commission (e.g., Michigan)
  - use of algorithms to detect gerrymandering
- Role of experts
  - legislative process
  - court testimony
- Open problems
  - large-scale redistricting problems (e.g., state legislatures)
  - algorithm-generated redistricting plan proposals
  - communities of interest, impact of redistricting rules